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Optimum and Suboptimum Design of FIR and SAW Filters: Remez Exchange Algorithm, LP, NLP, WLMS

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Part 1: Chebyshev (Minimax) Approximation Problem

Introduction

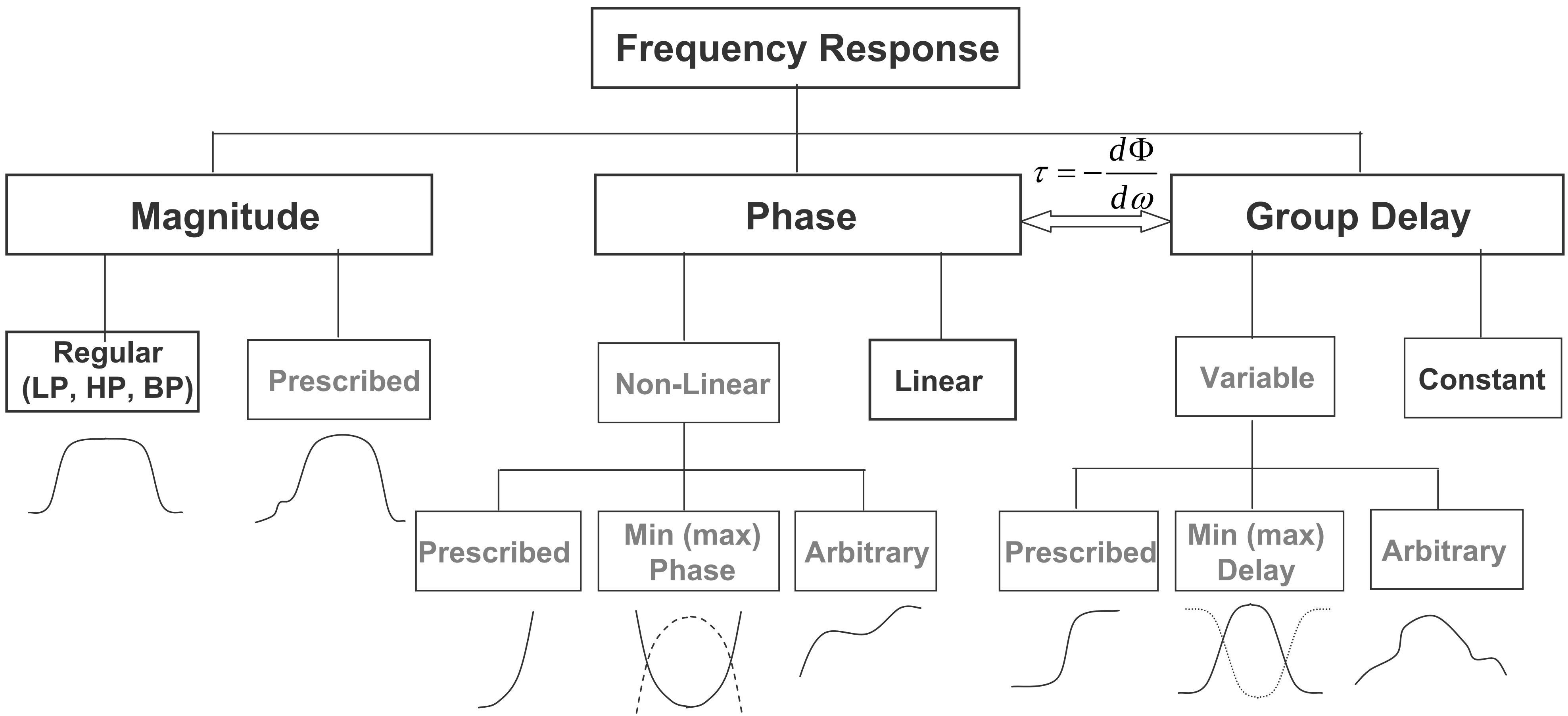
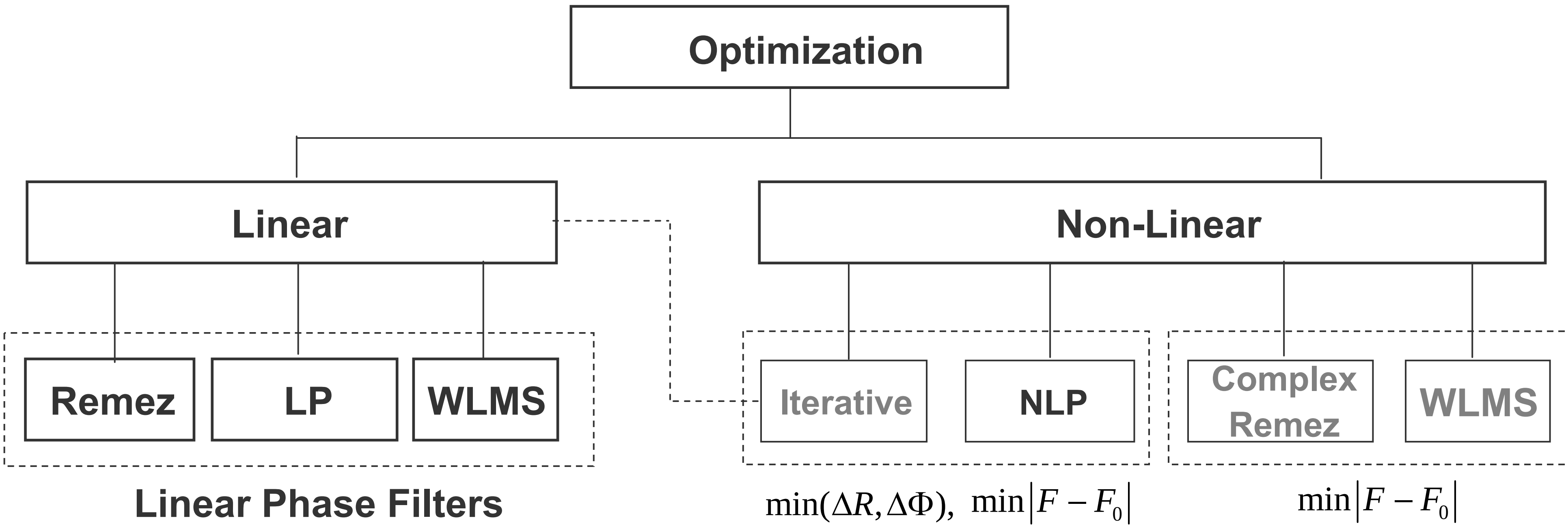


Fig. 1.1. Frequency response classification

Optimization Techniques

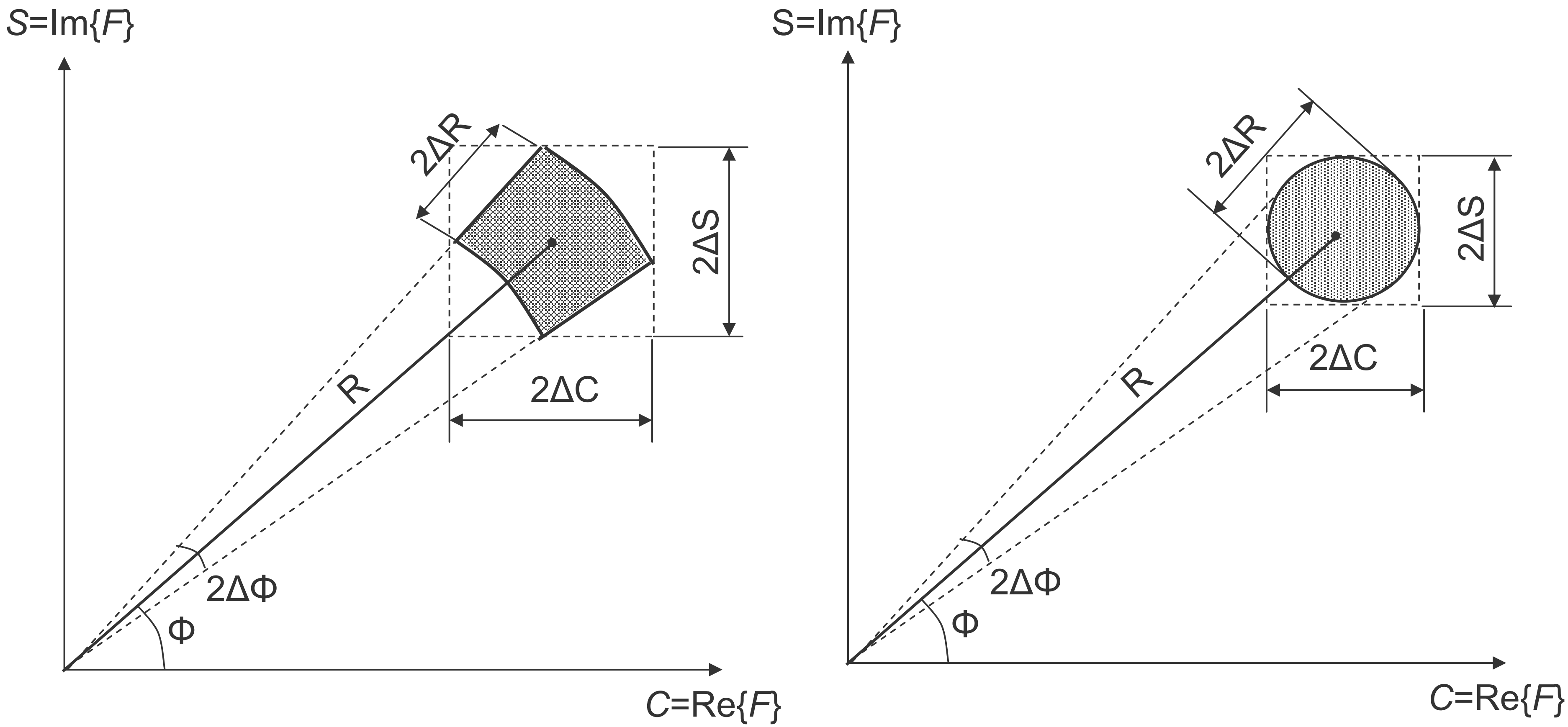


$$F = C + jS = \pm F^* = \begin{cases} \mathrm{Re}\{F\} = C \\ j\,\mathrm{Im}\{F\} = S \end{cases}$$

$$F = C + jS = R\,e^{j\Phi}$$

Fig. 1.2. Linear and non-linear optimization techniques

Different Tolerance Schemes



a) Non-linear tolerance field (ΔR, ΔΦ) b) Complex Chebychev error ΔR

Fig. 1.3. Non-linear phase response approximation

Features of Surface Acoustic Wave (SAW) Filters

SAW filters have the following features if compared to conventional non-recursive Finite Impulse Response (FIR) digital filters:

- SAW filters consist of two (or more) SAW transducers both contributing to an overall SAW filter response
- SAW filter frequency response is distorted by other frequency-dependent factors (element factor as the frequency-dependent acoustic source, multistrip coupler (MSC), etc.)
- Length of a narrowband SAW filter (< 1% passband width) may exceed thousands taps, with a large number of the small side-lobes to be reproduced technologically to a high accuracy
- SAW filter response is subjected to the distortion due to number of the second order effect (electrical source/load interaction, diffraction, finger resistance, etc.)

Basic Assumptions

1) The frequency response of the input SAW transducer is specified a priori, while the output SAW transducer is optimized to meet the specifications to the overall SAW filter response

2) To the first order, second-order effects can be neglected, i.e. SAW transducers can be approximated as FIR digital filters.

3) If we assume that the input transducer response is uniform at all frequencies (identically equal to one), the overall SAW filter response is equivalent to a FIR digital filter response.

Statement of the Problem

Consider a problem of the approximation of the prescribed real-valued function (target function) $D(\omega)$ by the function (frequency response)

$$F(\omega) = \sum_{k=0}^{n-1} a_k f_k(\omega) \qquad (1.1)$$

where $f_k(\omega)$ are the basis functions, a_k are the approximation coefficients.

At any frequency ω the approximation accuracy is characterized by the weighted Chebyshev error function

$$E(\omega) = W(\omega)\left[F(\omega) - D(\omega)\right] \qquad (1.2)$$

where $W(\omega) > 0$ is the weighting function.

Best (Minimax) Fit

The Chebyshev (minimax) approximation is the best fit to the desired real function $D(\omega)$ to minimize an absolute error

$$\delta = \min_{\mathbf{A}} \left\| E(\omega) \right\| = \min_{\mathbf{A}} \left\{ \max_{\omega \in \Omega} \left| E(\omega) \right| \right\} \qquad (1.3)$$

over a set of the coefficients $\mathbf{A}=[a_k]$ within the approximation interval Ω.

According to Eq. (1.3) the Chebyshev approximation problem is a linear *minimax* problem to minimize the maximum error over a specified frequency range.

Properties of Chebyshev Approximation

Alternation Theorem

Given the function $F(\omega)$ of the form (1.1) which is a linear combination of n basis functions $f_k(\omega)$ satisfying the Haar condition, a necessary and sufficient condition for this function to be the unique and best weighted-Chebyshev approximation to a continuous function $D(\omega)$ on a compact frequency subset Ω is that the weighted error function $E(\omega)$ exhibit at least $n+1$ extremal frequencies ω_i on Ω where the weighted error function takes equal and sign-alternating extremal values, i.e. there must exist $n+1$ frequency points $\omega_0 < \omega_1 < ... < \omega_n$ such that

$$E(\omega_i) = -E(\omega_{i+1}) = (-1)^i \delta \qquad (1.4)$$

Note. The Haar (or interpolation) property means that the coefficients a_k in Eq. (1.1) can be chosen such that the approximating function $F(\omega)$ is equal to zero at n+1 arbitrary prescribed frequency points ω_i within an interval Ω.

There exist several optimization techniques to solve the Chebyshev (minimax) approximation problem Eq. (1.1-1.3), some of them based on the equiripple behavior (1.4) of the optimal approximation.

Part 2: Chebyshev Approximation Techniques

Remez Exchange Algorithm (REA)

1. Given a set of the extremal frequencies ω_i from the previous iteration, a linear system of equations is constructed based on the alternation theorem

$$W(\omega_i)\left[\sum_{k=0}^{n-1} a_k f_k(\omega_i) - D(\omega_i)\right] = (-1)^i \delta, \quad i = 0, 1, \ldots, n-1 \qquad (2.1)$$

or in the matrix form

$$\begin{bmatrix} f_{0,0} & f_{0,1} & \cdots & f_{0,n-1} & \frac{+1}{W_0} \\ f_{1,0} & f_{1,1} & \cdots & f_{1,n-1} & \frac{-1}{W_0} \\ \vdots & & & \ddots & \vdots \\ f_{n-1,0} & f_{n-1,1} & \cdots & f_{n-1,n-1} & \frac{(-1)^{n-1}}{W_0} \\ f_{n,0} & f_{n,1} & \cdots & f_{n,n-1} & \frac{(-1)^n}{W_0} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \\ \delta \end{bmatrix} = \begin{bmatrix} D_0 \\ D_1 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix} \qquad (2.2)$$

where $f_{ik} = f_k(\omega_i), \quad W_i = W(\omega_i), \quad D_i = D(\omega_i)$

Remez Exchange Algorithm (Cont'd)

2. The coefficients a_k, k=0,1,.., n-1 and the current deviation δ at the extremal frequencies ω_l are determined from the solution of the linear system of equations (2.2).
3. Using the coefficients a_k for function interpolation between the extremal frequencies locate new extremal frequencies ω_i at which the error function (2.2) takes maximum values which are supposed to be equiripple for the next iteration.
4. Repeat iterations until the optimum equiripple solution is found to to a good accuracy.

5. The iterations are interrupted when one of the following criteria is satisfied:
   1) no more extremuma ω_i are to be refined, i.e. all the extremal frequencies positions ω_i are stable versus iterations,
   2) all the extremuma are equiripple to a prescribed accuracy

$$\frac{\max\left|E(\omega_i)\right| - \min\left|E(\omega_i)\right|}{\max\left|E(\omega_i)\right|} \leq \varepsilon \qquad (2.3)$$

REA Convergence

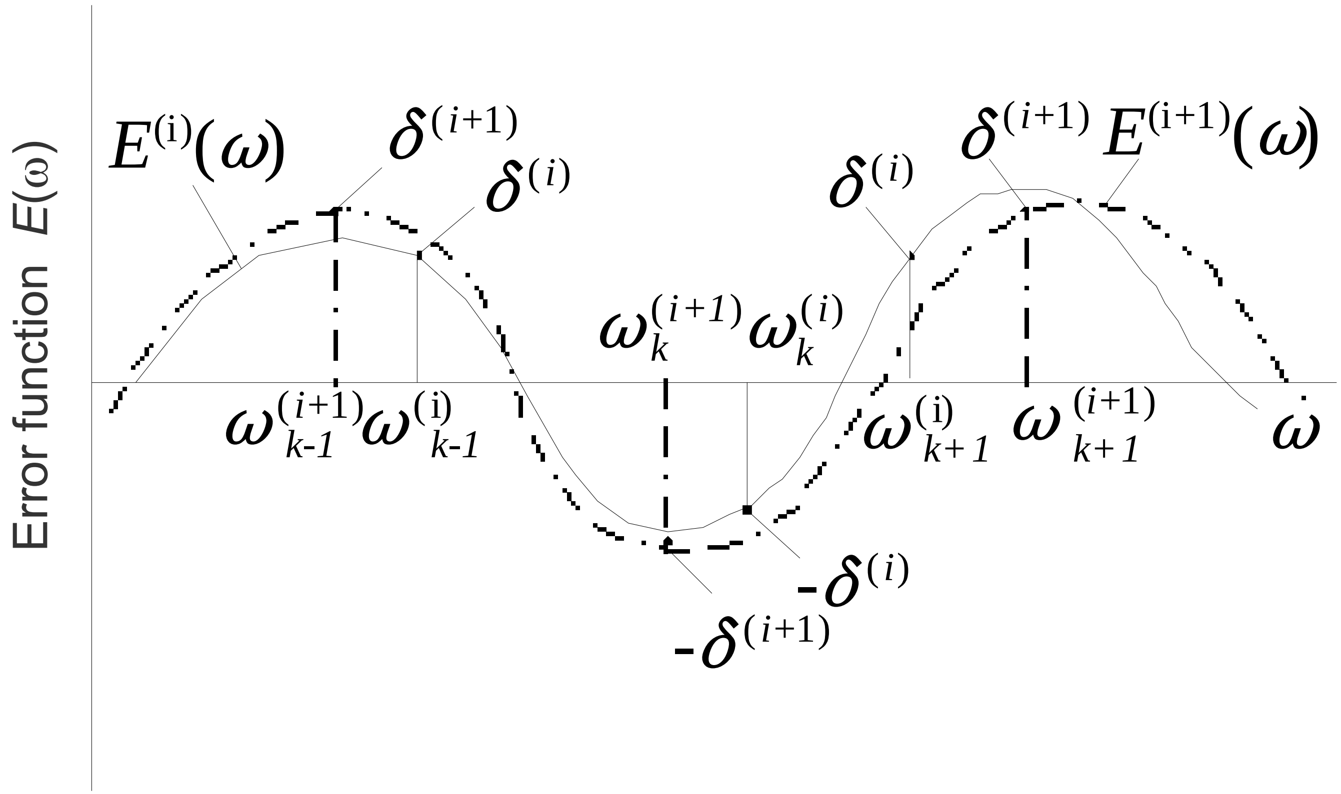


Fig. 2.1. Allocation of the extremal frequencies and interpolation scheme.
Remez exchange algorithm: —— *i*-th iteration, — · — *(i+1)*-th iteration

REA Implementation

Basic Algorithm Steps:

1) computation of the function $F(\omega_k)$ on the discrete frequency grid
2) search for the extremal frequencies ω_i
3) numerical solution of the system of linear equations (2.2) at each iteration to determine the deviation δ and coefficients a_k for a given set of the extremal frequencies ω_i.

For the acceptable accuracy a grid discretization interval $\delta\omega$ must satisfy the condition $\delta\omega/\Delta\omega \approx 8\text{-}10$ where $\Delta\omega=\omega s/N$ is the discrete Fourier transform (DFT) interval, ωs is the sampling frequency.

Basis Conversion

$$P(\varphi) = \sum_{k=0}^{n-1} a_k \cos k\varphi \quad \xrightarrow{x = \cos\varphi} \quad P(x) = \sum_{k=0}^{n-1} a_k x^k \tag{2.4}$$

trigonometric → algebraic

where $\varphi = \pi\omega/\omega_\pi$, $\omega_\pi = \omega_s/2$ is synchronous frequency (half-sampling frequency).

Algebraic basis matrix $F=[f_{ik}]=x_i^k$ has a Vandermonde structure

$$F = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-1} \\ \vdots & & & \ddots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^{n-1} \end{bmatrix} \tag{2.5}$$

Closed-form solution of the system (2.2):

$$\delta = \frac{\sum_{k=0}^{n} \alpha_k D_k}{\sum_{k=0}^{n} (-1)^k \alpha_k W_k}, \quad \alpha_k = \prod_{i \neq k} (x_k - x_i) \tag{2.6}$$

Lagrange Interpolation

The Lagrange interpolation formula can be applied to evaluate polynomial values on the frequency grid which can be written in the baricentric form [1]

$$\delta = \frac{\sum_{k=0}^{n} \frac{\alpha_k}{x - x_k}}{\sum_{k=0}^{n} \frac{\alpha_k}{x - x_k} y_k}, \quad x_k = \cos \pi \frac{\omega_k}{\omega_\pi}, \quad y_k = D_k + (-1)^k \frac{\delta}{W_k} \qquad (2.7)$$

1. The closed-form solution (2.7) makes the Remez exchange algorithm very effective for practical Chebyshev approximation using the trigonometric basis.

2. A very reliable McClellan's computer FORTRAN program [2] is available for designing finite impulse response (FIR) nonrecursive digital filters which makes the basis for many applications where the Chebyshev approximation is required.

Linear Programming (LP)

Statement of the LP Problem

$$\min_{\mathbf{X}} \mathbf{C}^T\mathbf{X} - ? \qquad (2.8)$$

$$\mathbf{AX} \leq \mathbf{B} \qquad (2.9)$$

$$\mathbf{A} = [a_{\mathrm{ik}}], \quad i \leq m, k \leq n$$

$$\mathbf{X} = [x_1 \quad x_2 \quad \ldots \quad x_n]$$

$$\mathbf{B} = [b_1 \quad b_2 \quad \ldots \quad b_m]$$

$$\mathbf{C} = [c_1 \quad c_2 \quad \ldots \quad c_n]$$

X vector of variables (unknowns)

A matrix of the constraint coefficients of size *m x n*

C vector of the coefficients of the linear objective (goal) function

m is the number of linear constraints (inequalities or equalities)

n is the number of variables

Conversion of Minimax Problem to LP Form

$$\min_{A} \delta - ? \tag{2.10}$$

$$|E(\omega_i)| \le \delta, \quad \omega_i \in \Omega \tag{2.11}$$

where $E(\omega_i)= W(\omega_i)[F(\omega_i)-D(\omega_i)]$ is the weighted Chebyshev approximation error specified on a discrete set of frequencies (frequency grid) $\omega_i \in \Omega$.

By using the notation $E_i=E_i(\omega)$, $W_i=W_i(\omega)$, $D_i=D_i(\omega)$, $f_{ik}=f_k(\omega_i)$ we can rewrite the constraints (2.11) as

$$\begin{cases} \sum_{k=1}^{n} f_{ik} a_k - \dfrac{\delta}{W_i} \le D_i \\ -\sum_{k=1}^{n} f_{ik} a_k - \dfrac{\delta}{W_i} \le -D_i \end{cases} \tag{2.12}$$

Augment the vector of the variables **A** to include an additional variable δ and the matrix **F**=[f_{ik}] to include the column ±1/W_i

$$a_{n+1} = \delta, \quad f_{ik}^{\pm} = \begin{cases} f_{ik}, \ k \le n \\ \pm\dfrac{1}{W_i}, \ k = n+1 \end{cases} \tag{2.13}$$

Discrete Chebyshev Approximation

$$\min_{\mathbf{A}} \mathbf{C}^T \mathbf{A} - ? \qquad (2.14)$$

$$\begin{cases} \mathbf{F}^{-}\mathbf{A} \le +\mathbf{D} \\ -\mathbf{F}^{+}\mathbf{A} \le -\mathbf{D} \end{cases} \qquad (2.15)$$

$\mathbf{C} = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \end{bmatrix}^T$ vector of the objective function coefficients

$\mathbf{A} = \begin{bmatrix} a_1 & a_2 & \dots & a_n & \delta \end{bmatrix}^T$ augmented vector of the coefficients

$\mathbf{F}^{\pm} = \left[f_{ik}^{\pm} \right], i \le m, k \le n+1$ augmented matrices of the basis functions values on the frequency grid

$\mathbf{D} = \begin{bmatrix} D_1 & D_2 & \dots & D_m \end{bmatrix}^T$ vector of the desired (target) function values on the grid

The discrete Chebyshev approximation problem can be converted to the standard LP problem and solved by LP techniques, for example, by the simplex method.

Merits of Linear Programming

1) The basis functions may *not* satisfy the Haar and/or orthogonality conditions

2) Additional linear constraints can be implied:

- on the function $F(\omega)$ in the frequency or time domain
- on the coefficients a_k (or any linear combination of them), e.g. upper and lower bounds

Constraints Examples

1) An accurate interpolation of the desired function $D(\omega)$) at some frequency points ω_i, i.e. the error function $E(\omega_i)=0$ at these frequencies.

2) All or some of the coefficients a_k have the bounded *min* and *max* values

$$a_{\min} \leq a_k \leq a_{\max} \tag{2.16}$$

3) Some coefficients a_k take pre-specified values (say, zero) (e.g. Nyquist filters where each *m*-th time sample must vanish for good suppression of the intersymbol interference).

LP Design Summary

1. LP is a very powerful and flexible design tool for linear-phase SAW filter synthesis.

2. LP is intrinsically slower if compared to the Remez exchange algorithm and may become impractical as the number of the optimized variables increases.

3. The parameter reduction schemes must be applied based on the resampling (decimation) techniques in the frequency or time domain.

4. The Remez exchange algorithm and LP are sophisticated optimization tools which are not easy for programming. The standard commercial software implementing both techniques is available.

Weighted Least Mean Squares (WLMS)

Advantages of WLMS

1. The WLS solution is well known for a long time, with the matrix closed-form solution for the *prescribed* least-squares weighting function.

2. The algorithm is easy for programming and many commercial LMS programs exist.

3. A successful WLMS implementation producing the equiripple (minimax) design depends on the appropriate weighting least-squares scheme.

4. Several enhanced iterative techniques to derive the weighting function at each iteration have been developed [3-8].

WLMS Problem

Given a real-valued approximating function $F(\omega)$, desired (target) function $D(\omega)$, and Chebyshev weight function $W(\omega)$, the weighted least-squared error on the discrete frequency grid ω_i, i=0, m-1 is

$$\chi^2 = \sum_{i=0}^{m-1} w_i^2 \left[F_i - D_i\right]^2 = \sum_{i=0}^{m-1} w_i^2 \left[\sum_{k-0}^{n-1} a_k f_{ik} - D_i\right]^2 \qquad (2.17)$$

where w_i are the weights in the LMS error function, F_i, D_i, f_{ik} as defined earlier.

$$\min_{\mathbf{A}} \chi^2 - ?$$

WLMS Matrix Form

$$\chi^2 = (\mathbf{FA} - \mathbf{D})^T \mathbf{W}^2 (\mathbf{FA} - \mathbf{D}) \qquad (2.18)$$

$$\mathbf{F} = \begin{bmatrix} f_{0,0} & f_{0,1} & \cdots & f_{0,n-1} \\ f_{1,0} & f_{1,1} & \cdots & f_{1,n-1} \\ \vdots & & \ddots & \vdots \\ f_{m-1,0} & f_{m-1,1} & & f_{m-1,n-1} \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} w_o & 0 & \dots & 0 \\ 0 & w_1 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & w_{m-1} \end{bmatrix} \qquad (2.19)$$

$$\mathbf{A} = \begin{bmatrix} a_0 & a_1 & \dots & a_{n-1} \end{bmatrix}^T, \quad \mathbf{D} = \begin{bmatrix} D_0 & D_1 & \dots & D_{m-1} \end{bmatrix}$$

Given the functions $D(\omega)$, $F(\omega)$ and $W(\omega)$ on the discrete frequency grid ω_i, i=0, m-1, the problem is minimizing the weighted least squared error (2.18) over a set of the coefficients a_k.

Closed-Form WLMS Solution

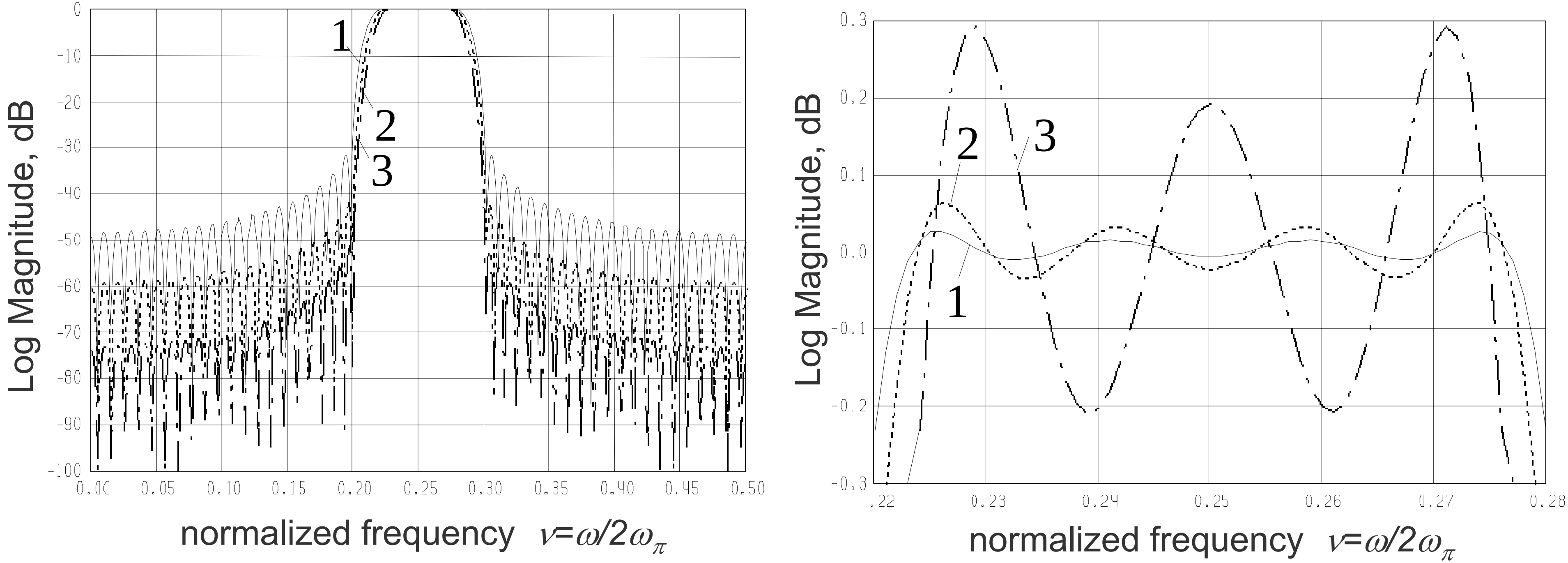
$$\frac{d\chi^2}{dA} = (\mathbf{FA} - \mathbf{D})^T \mathbf{W}^2\mathbf{F} = 0 \tag{2.20}$$

The closed-form solution of Eq. (2.20):

$$\mathbf{A} = (\mathbf{F}^T \mathbf{W}^2\mathbf{F})^{-1}(\mathbf{F}^T \mathbf{W}^2\mathbf{D}) \tag{2.21}$$

The complexity of Eq. (2.21) is defined by matrix multiplications and matrix inversion. Linear algebra techniques and commercial computer programs can be applied to find the optimum LMS solution.

Example of Non-Iterative WLMS Filter Design

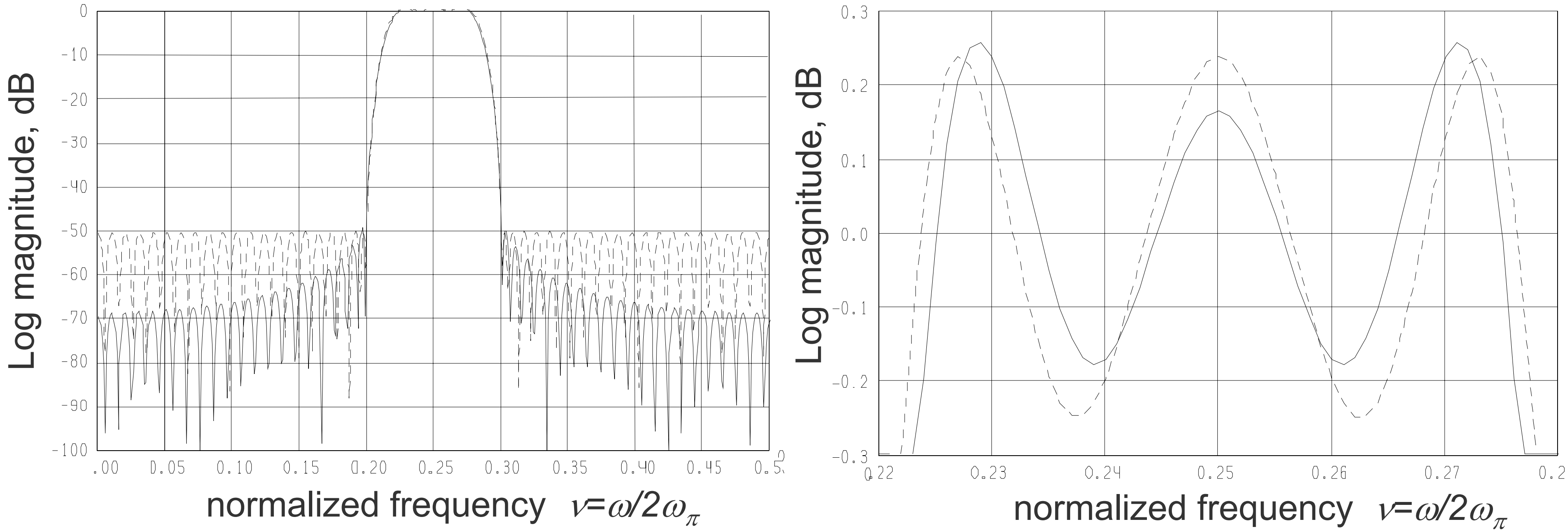


a) magnitude response

b) passband ripple

Fig. 2.2. WLS approximation (N=97) for different weighting passband/stopband ratios. Filter specifications: SB: [0, 0.2] ∩ [0.3, 0.5], PB: [0.2225, 0.2775].
(1, 2, 3 – w_{pb}/w_{sb}=10, 1, 0.1)

Comparison with Optimum (Chebyshev) Approximation



a) magnitude response

b) passband ripple

Fig. 2.3. Comparison of the LMS and optimum (Chebyshev) solutions (w_{pb}/w_{sb}=8):
—— LMS, - - - Chebyshev

1. Difference between the WLMS solution and Chebyshev approximation can be minimized by the appropriate choice of the stopband-to-passband weight ratio.
2. The WLMS solution can be further improved by individual adjusting the LMS weights to reduce approximation error at the expense of the “over-approximated” regions.

WLMS Convergence

The weighting least-squared function $w(\omega)$ can be redefined at each k-th iteration as follows

$$w_{k+1}(\omega) = \xi_k^{\theta}(\omega) w_k(\omega) \qquad (2.22)$$

where $\xi_k(\omega)$ is the weight correction (update) function and θ is the empirical convergence factor.

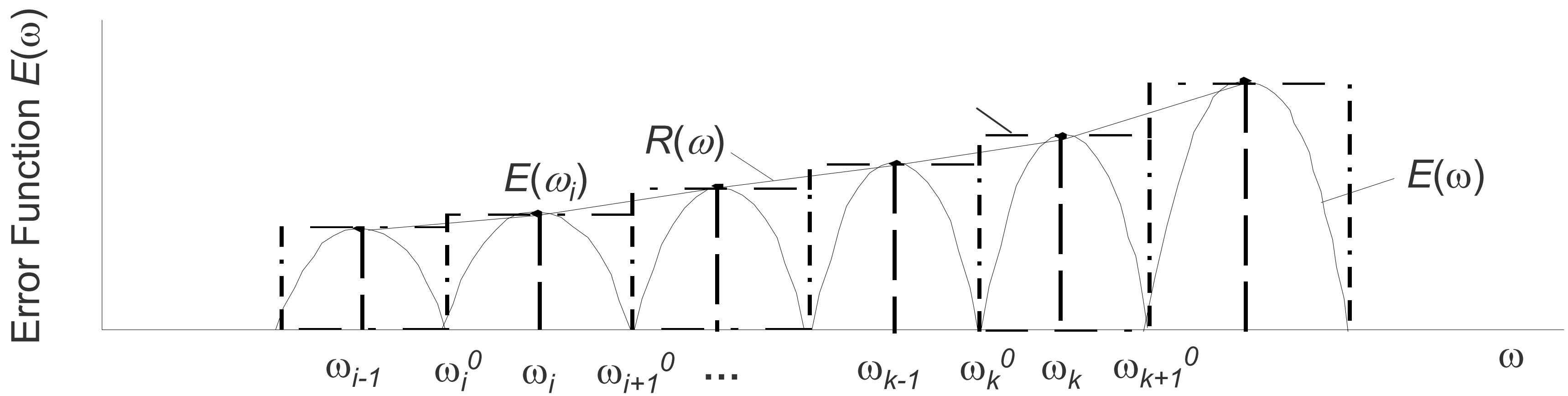


Fig. 2.4. WLMS approximations of the Chebyshev error function

Reweighting Iterative Schemes

1. Lawson's algorithm [5]

$$\xi_k(\omega) = E_k(\omega) \quad (2.23)$$

where $E_k(\omega)=|F_k(\omega)-D(\omega)|$ is the absolute Chebyshev error function.

2. Step-wise error approximation [6]

$$\xi_k(\omega) = \max\{E(\omega)\} = E(\omega_i), \quad \omega_i^0 \le \omega \le \omega_{i+1}^0 \quad (2.24)$$

where ω_i is the position of maximum, ω_i^0, ω_{i+1}^0 are the positions of minimum (valley frequencies)

3. Envelope approximation [7]

$$\xi_k(\omega) = R_k(\omega) \quad (2.25)$$

where $R_k(\omega)$ is the error envelope function.

WLMS Convergence (Stop Band)

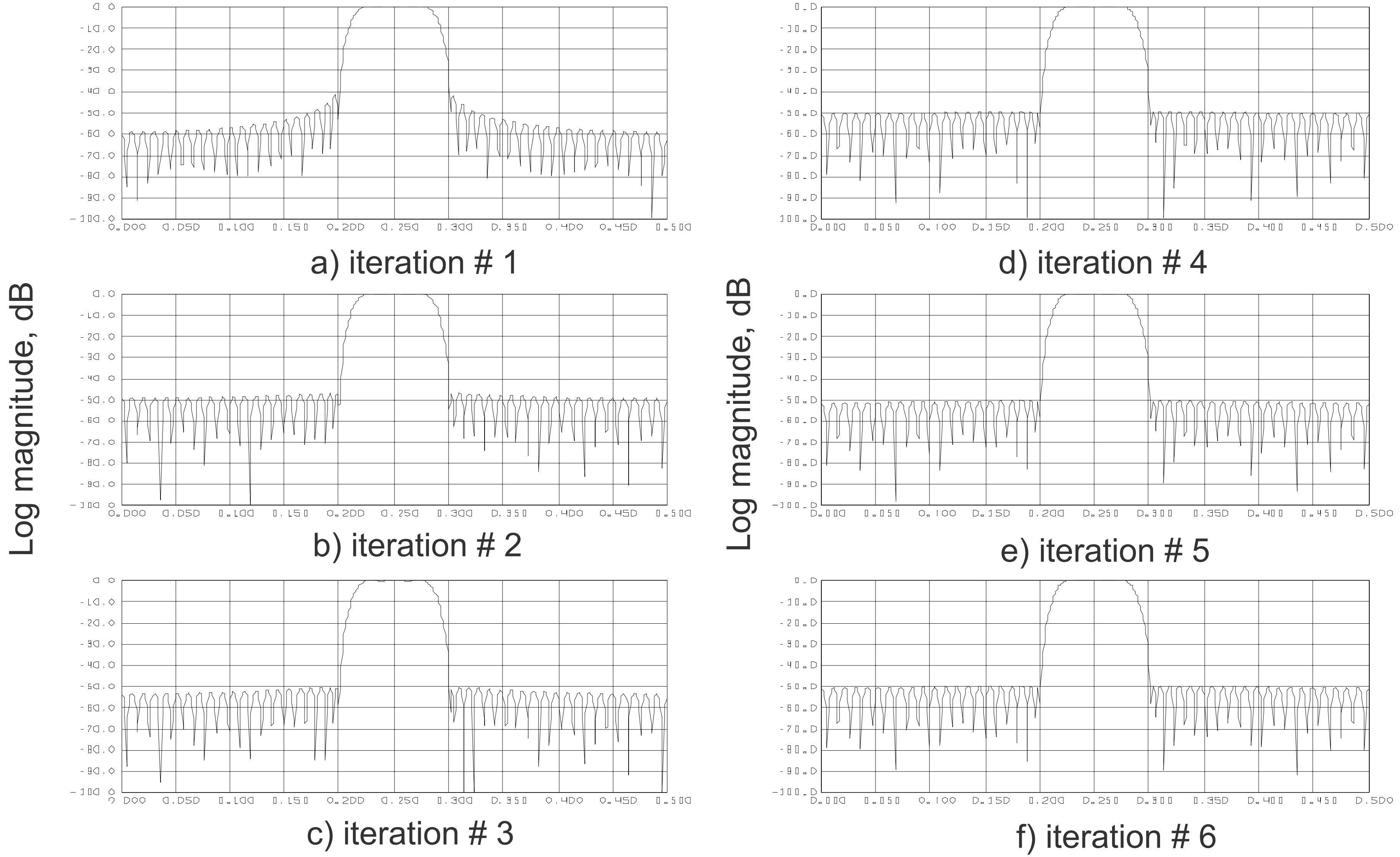


Fig. 2.5. Quasi-equiripple WLMS approximation (stop band, log scale)

WLMS Convergence (Pass Band)

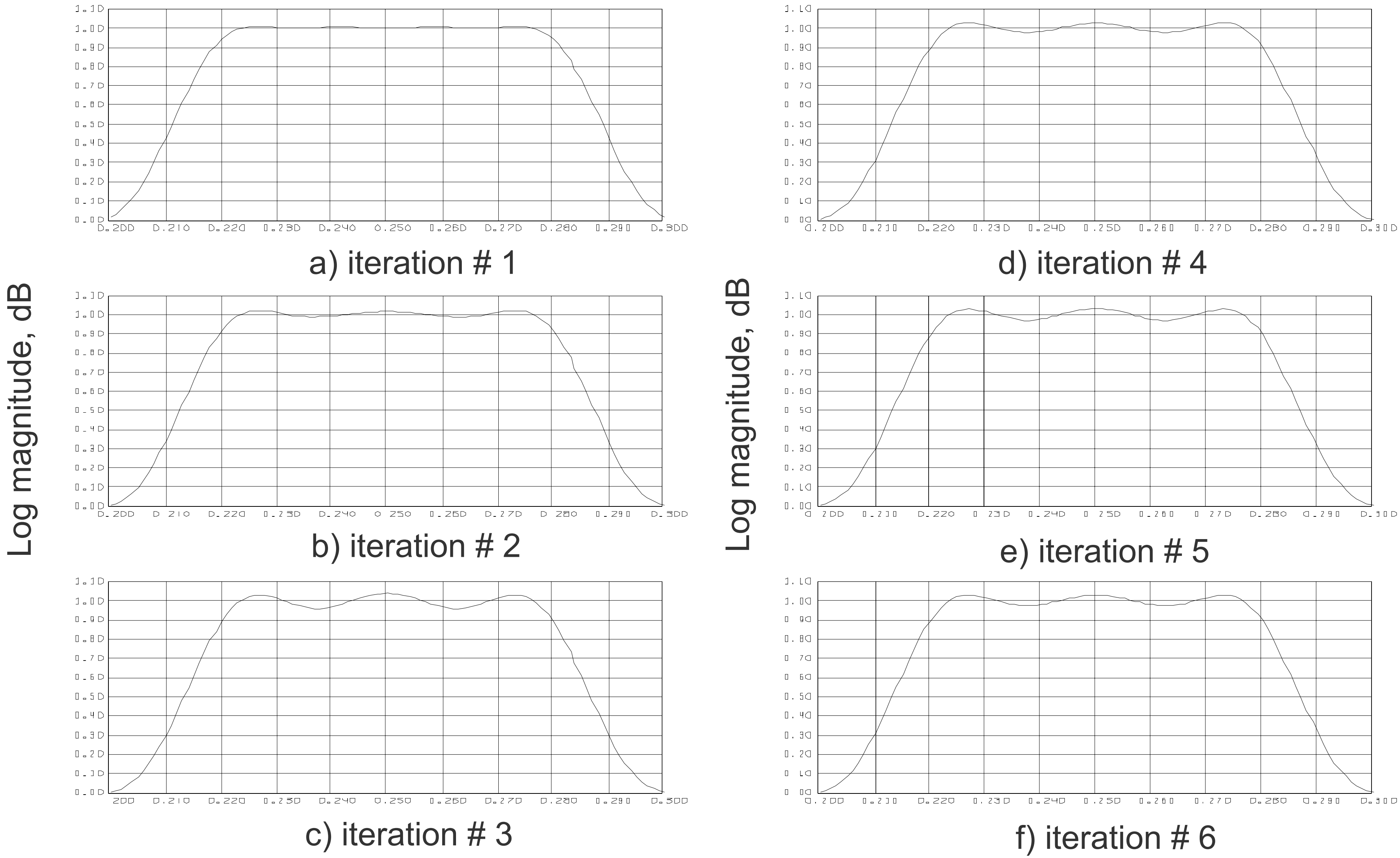


Fig. 2.6. Quasi-equiripple WLMS approximation (pass band, linear scale)

Example (WLMS): FIR Digital Filter

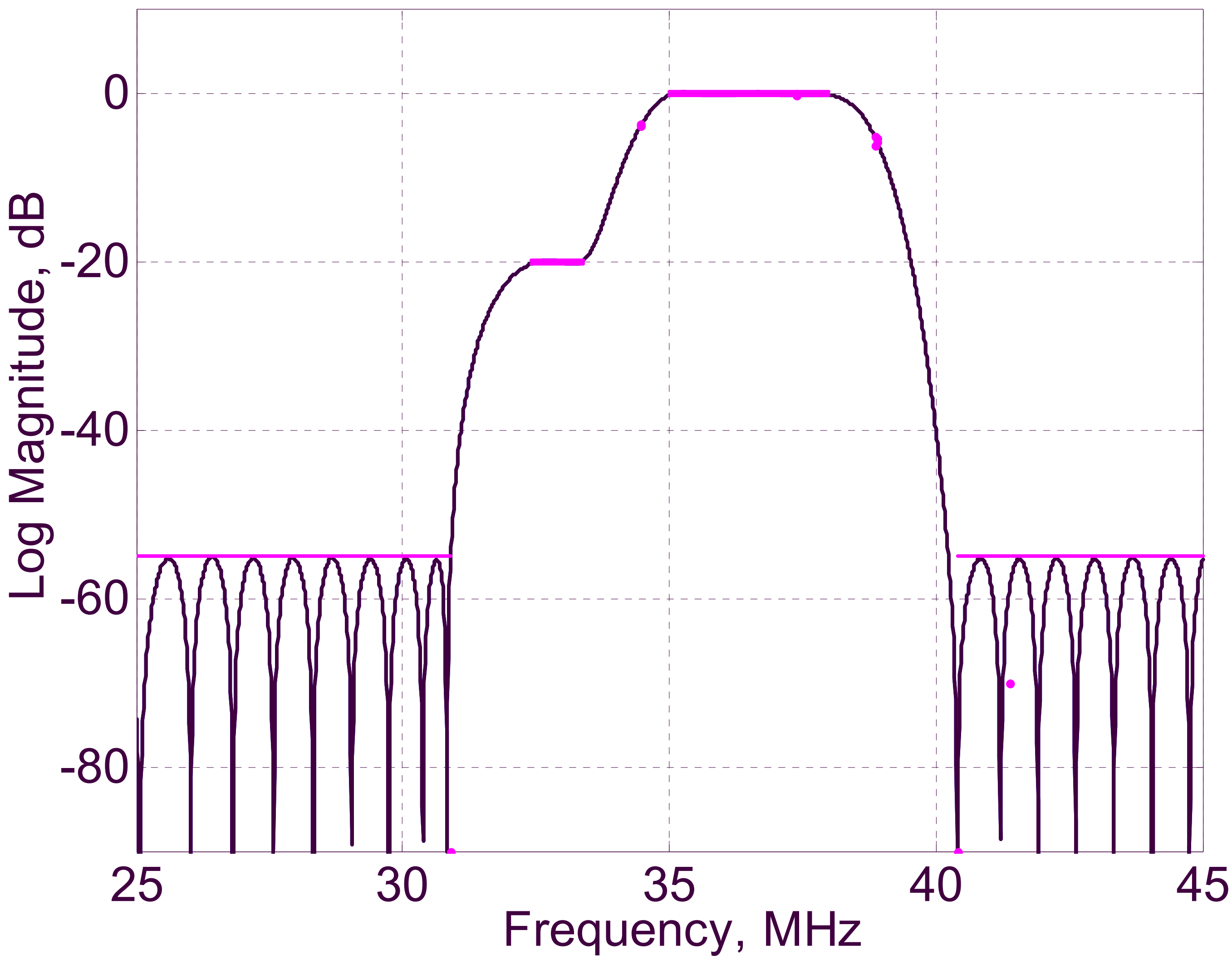


Fig. 2.7. FIR Digital Filter with prescribed magnitude response, *N*=192

Example (WMLS): FIR Digital Filter (Pass Band)

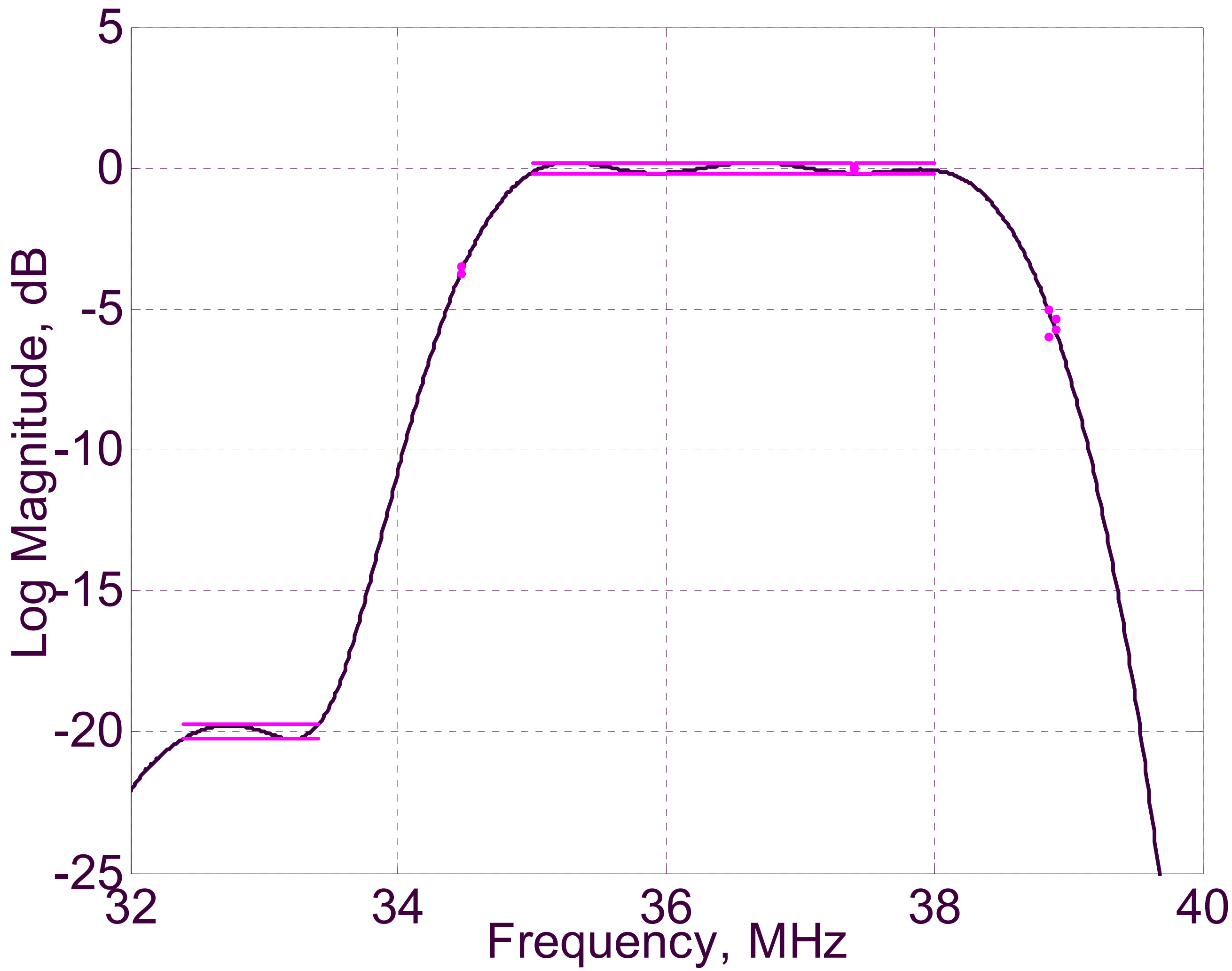


Fig. 2.8. FIR Digital Filter with prescribed magnitude response (Pass Band), N=192

Example (WLMS): FIR Digital Filter (Tap Weights)

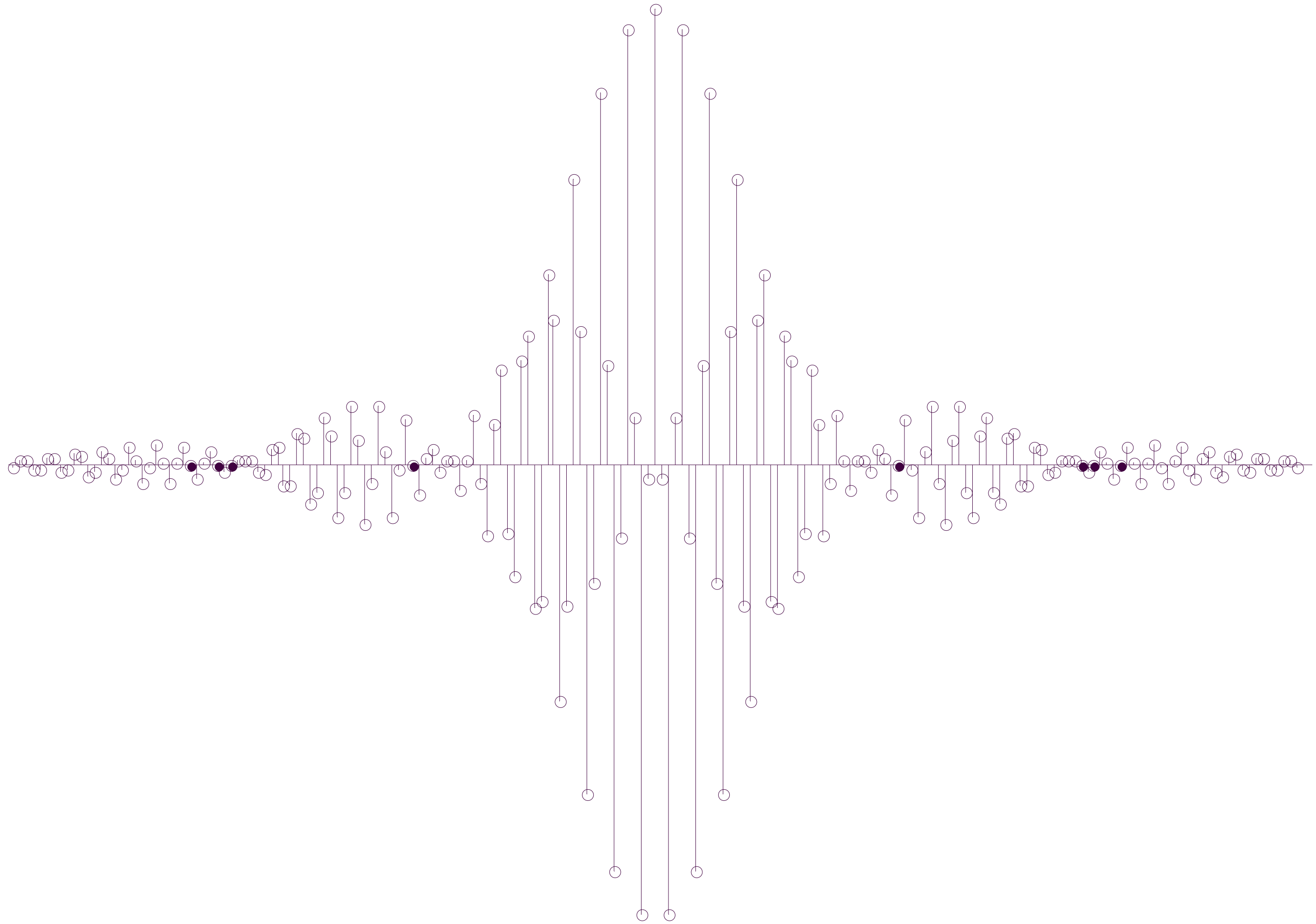


Fig. 2.9. FIR Digital Filter tap weights, N=192

Part 3: Non-Linear Phase Design

Complex WLMS (CWLMS)

Chebyshev Approximation to Specified Complex Function

Complex Chebyshev error function

$$E(\omega) = W(\omega)\left[F(\omega) - D(\omega)\right] \tag{3.1}$$

$F_0(\omega) = R_0(\omega)e^{j\theta_0(\omega)}$ the specified (target) complex-valued response

$F(\omega) = F(\mathbf{A}, \omega) = R(\omega)e^{j\theta(\omega)}$ the approximating function that is linear w.r.t. a vector **A** of the *n* variables to be optimized

$F(\omega) = C(\omega) + jS(\omega)$ complex frequency response

$R(\omega) = \left|F(\omega)\right| = \sqrt{C^2(\omega) + S^2(\omega)}$ magnitude response

$\theta(\omega) = \arg\left\{F(\omega)\right\} = \arctan\dfrac{S(\omega)}{C(\omega)}$ phase response

$$\delta = \min_{\mathbf{A}} \left\|E(\omega)\right\| = \min_{\mathbf{A}} \left\{ \max_{\omega \in \Omega} \left|E(\omega)\right| \right\} \tag{3.2}$$

Graphical Error Representation in Complex Domain

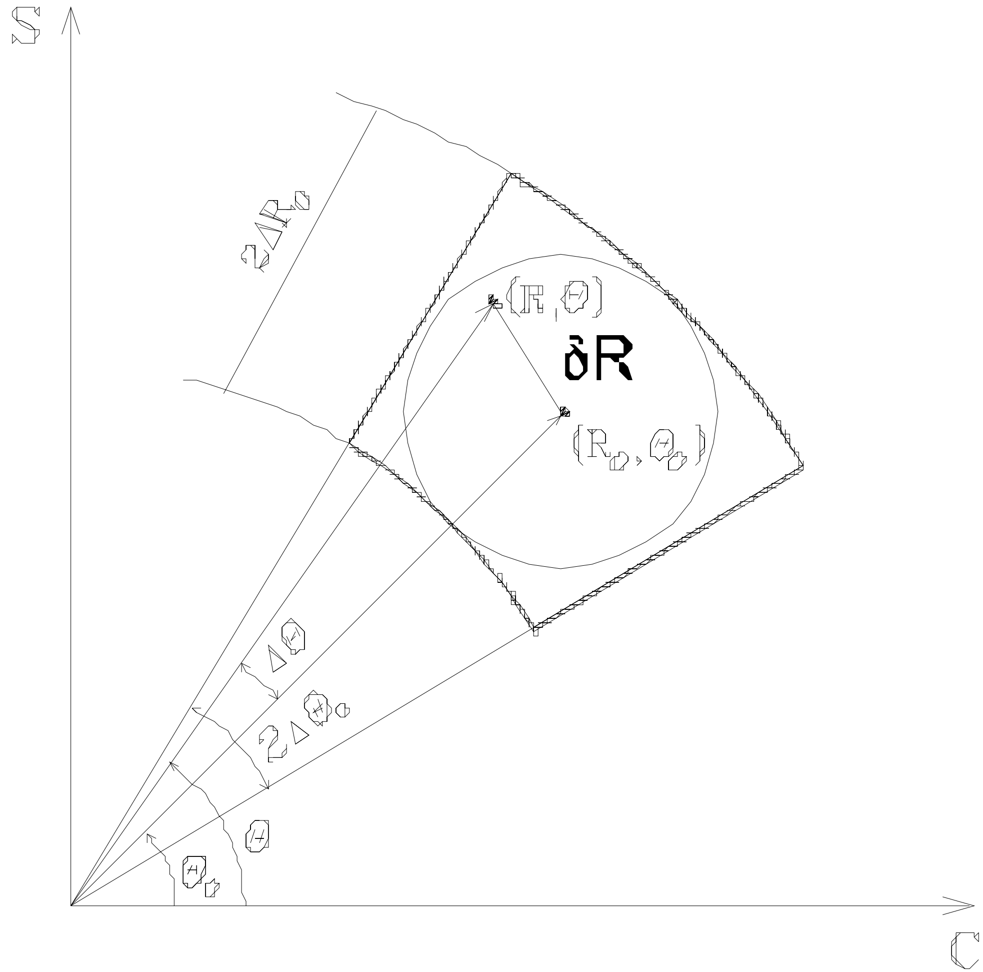


Fig. 3.3. Complex Chebyshev error function

CWLMS Problem

Given a real-valued approximation function

$$F(\omega) = \sum_{k-0}^{n-1} a_k C_k(\omega) + j \sum_{k-0}^{n-1} b_k S_k(\omega) \qquad (3.4)$$

where $C_k(\omega)$ and $S_k(\omega)$ are the real and imaginary part basis functions, desired (target) function $D(\omega)$, and Chebyshev weight function $W(\omega)$, the weighted least-squared error on the discrete frequency grid ω_i, i=0, m-1 is

$$\chi^2 = \sum_{i=0}^{m-1} w_i^2 \left[F_i - D_i\right]^2 = \sum_{i=0}^{m-1} w_i^2 \left[\sum_{k-0}^{n-1} a_k C_{ik} + j \sum_{k-0}^{n-1} b_k S_{ik} - D_i\right]^2 \qquad (3.5)$$

where w_i are the weights in the LMS error function, F_i, D_i, as defined previously, C_{ik}=$C_k(\omega_i)$ and S_{ik}=$S_k(\omega_i)$ are basis functions of the real and imaginary components of the approximating function $F(\omega_i)$ on the discrete frequency grid ω_i.

CWLMS Matrix Form

$$\chi^2 = (\mathbf{FA} - \mathbf{D})^* \mathbf{W}^2 (\mathbf{FA} - \mathbf{D}) \qquad (3.6)$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{C} & j\mathbf{S} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} C_{0,0} & C_{0,1} & \cdots & C_{0,n-1} \\ C_{1,0} & C_{1,1} & \cdots & C_{1,n-1} \\ \vdots & & \ddots & \vdots \\ C_{m-1,0} & C_{m-1,1} & & C_{m-1,n-1} \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} S_{0,0} & S_{0,1} & \cdots & S_{0,n-1} \\ S_{1,0} & S_{1,1} & \cdots & S_{1,n-1} \\ \vdots & & \ddots & \vdots \\ S_{m-1,0} & S_{m-1,1} & & S_{m-1,n-1} \end{bmatrix} \qquad (3.7)$$

$$\mathbf{a} = \begin{bmatrix} a_0 & a_1 & \ldots & a_{n-1} \end{bmatrix}^T, \quad b = \begin{bmatrix} b_0 & b_1 & \ldots & b_{n-1} \end{bmatrix}^T$$

$$\mathbf{D} = \begin{bmatrix} D_0 & D_1 & \ldots & D_{m-1} \end{bmatrix}, \quad \mathbf{W} = diag\{\begin{bmatrix} w_0 & w_1 & \ldots & w_{m-1} \end{bmatrix}\}$$

* denotes Hermitian conjugation.

Given the functions $D(\omega)$, $F(\omega)$ and $W(\omega)$ on the discrete frequency grid ω_i, i=0, m-1, the problem is minimizing the weighted least squared error (3.6) over a set of the coefficients $\{a_k , b_k\}$.

CWLMS Solution

$$\frac{d\chi^2}{dA} = (\mathbf{FA} - \mathbf{D})^* \mathbf{W}^2 \mathbf{F} = 0 \qquad (3.8)$$

By splitting Eq. (3.8) into real and imaginary parts and substituting **A=[a b]**^{T}

$$\begin{cases} (\mathbf{Ca} - \mathrm{Re}\,\mathbf{D})^T \mathbf{W}^2 \mathbf{C} = 0 \\ (\mathbf{Sb} - \mathrm{Im}\,\mathbf{D})^T \mathbf{W}^2 \mathbf{S} = 0 \end{cases} \qquad (3.9)$$

The closed-form solution of Eq. (3.9) is given by

$$\begin{aligned} \mathbf{a} &= (\mathbf{C}^T \mathbf{W}^2 \mathbf{C})^{-1} \mathbf{C}^T \mathbf{W}^2 \,\mathrm{Re}\,\mathbf{D} \\ \mathbf{b} &= (\mathbf{S}^T \mathbf{W}^2 \mathbf{S})^{-1} \mathbf{S}^T \mathbf{W}^2 \,\mathrm{Im}\,\mathbf{D} \end{aligned} \qquad (3.10)$$

Therefore, complex-valued LMS problem is reduced to the two separate real-valued LMS problems for real and imaginary parts of $D(\omega)$.

Any of the reweighting schemes (2.23-2.25) can be applied to LMS weighting at each Iteration.

CWLMS Example: FIR Allpass Filter with Twin Group Delay

Non-Linear Programming

Chebyshev Approximation to Specified Magnitude and Phase/Group Delay

Given a desired (target) bandpass frequency response

$$F_0(\omega) = R_0(\omega)e^{j\theta_0(\omega)} \tag{2.26}$$

where $R_0(\omega)$ is the magnitude response and $\theta_0(\omega)$ is the phase response, we define the weighted Chebyshev magnitude, phase and group delay error functions

$$\begin{aligned} E_R(\omega) &= W_R(\omega)\left[F(\omega) - R_0(\omega)\right] \\ E_\theta(\omega) &= W_\theta(\omega)\left[\theta(\omega) - \theta_0(\omega)\right] \\ E_\tau(\omega) &= W_\tau(\omega)\left[\tau(\omega) - \tau_0(\omega)\right] \end{aligned} \tag{2.27}$$

$F(\omega) = F(\mathbf{A}, \omega)$ approximation function that is non-linear with respect to a vector **A** of the *n* variables to be optimized

$\tau(\omega) = -\dfrac{d\theta}{d\omega}$ group delay response

Non-Linear Chebyshev Approximation Problem

Feature: Multiobjective Goal Attainment Problem

$$\delta = \min_{\mathbf{A}} \left\{ \max_{\Omega}(|E_R(\omega)|, \ \max_{\Omega}(|E_\theta(\omega)|), \ \max_{\Omega}(|E_\tau(\omega)|) \right\} \quad - \quad ? \qquad (2.28)$$

$$G_i(\mathbf{A}) \leq 0, \quad i = \overline{1, M} \qquad (2.29)$$

where $\mathbf{A} = \begin{bmatrix} a_1 & a_2 & \ldots & a_N \end{bmatrix}^{\mathbf{T}}$ is a vector of the optimized variables.

Goal Attainment Methods: Weighting

1. Weighted Sum Optimization

$$\delta = \min_{\mathbf{A}} \left\{ w_R \max_{\Omega} \left(\left| E_R(\omega) \right| + w_\theta \; \left| E_\theta(\omega) \right| + w_\tau \; \left| E_\tau(\omega) \right| \right\} \quad - \quad ? \quad (2.30)$$

where w_R, w_θ, w_τ are the fixed goal function weights to be specified a priori

Difficulty: How to choose the weights w_R, w_θ, w_τ ?

Technique: Trial and error

Goal Attainment Methods: Constraints

2. Constraints Method

$$\delta = \min_{\mathbf{A}} \left\{ \max_{\Omega} (|E_R(\omega)|) \right\} \quad - \quad ? \qquad (2.31$$

$$|E_\theta(\omega)| \le \Delta\theta$$
$$|E_\tau(\omega)| \le \Delta\tau \qquad (2.32)$$

where *ΔΘ, Δτ* are the fixed phase and group delay tolerances to be specified a priori.

Difficulty: How to find an optimum solution?

Goal Attainment Methods: Squeezing

3. Squeezing Method

$$\delta = \min_{\mathbf{A}} \lambda \quad - \quad ? \tag{2.33}$$

$$\left|E_R(\omega)\right| \le \lambda \Delta R$$

$$\left|E_\theta(\omega)\right| \le \lambda \Delta \theta \tag{2.34}$$

$$\left|E_\tau(\omega)\right| \le \lambda \Delta \tau$$

where $\lambda \le 1$ is a “squeezing” factor to control approximation accuracy

Difficulty: Simultaneous squeezing of all three tolerances.
Individual squeezing causes the same problem as the weighting technique: how to choose the weights?

Goal Attainment Methods: Chebychev Error Generalization

4. Generalized Chebychev Error

$$\delta = \min_{\mathbf{A}} \left\{ \max_{\Omega} \left(\left| E_R(\omega) \right|, \left| E_\theta(\omega) \right|, \left| E_\tau(\omega) \right| \right) \right\} \quad - \quad ? \qquad (2.35)$$

Particular Case: Linear Phase Optimization

$$\delta = \min_{\mathbf{A}} \left\{ \max_{\Omega} \left| E_R(\omega) \right| \right\} \quad - \quad ? \qquad (2.36)$$

$F(\omega) = F(\mathbf{A}, \omega)$ approximation function that is *sine* or *cosine* trigonometric polynomial w.r.t. the coefficients A to be optimized

NLP Troubleshooting

1. Good initial guess required.
2. Slow convergence and computational speed.
3. Local optima problem (global optimization).
4. Sensitivity to optimization parameters (termination tolerance, number of iterations, minimum/maximum step, etc.

Computational Speed of Optimization Algorithms

Test Filter Specifications

The test bandpass filter has the following frequency specifications in terms of the normalized frequency $\nu = \omega / 2\omega_{\pi}$:

passband range	[0.14-0.2]
stopband range	[0,0.12] W [0.22-0.5]
passband weight (Chebyshev)	1
stopband weight (Chebyshev)	1/5

The number of frequencies in the discrete grid:

$32n$ Remez exchange algorithm and LP ($n=[N/2]+1$)

$8n$ WLMS.

Comparison of Computation Time (Linear Optimization)

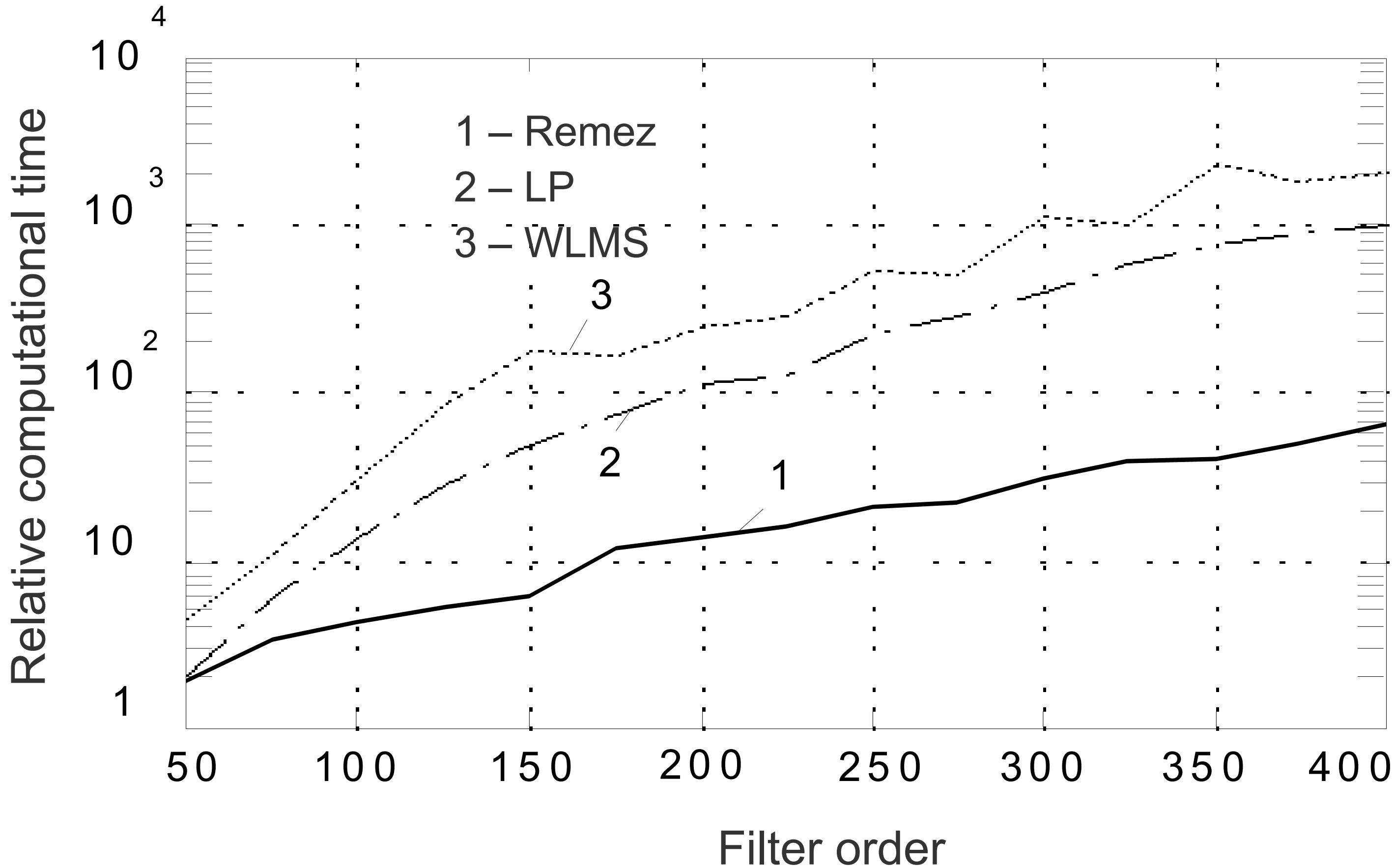


Fig. 3.4. Relative computational speed of the different optimization algorithms (after M.L.Matschesi et al. [3])

Test Summary

1. The Remez exchange algorithm is much superior over LP and WLMS w.r,t . speed.
2. The gain in computational speed achieves a factor of more than ten for high-order filters.
3. The slowest algorithm is the WLS algorithm which may be several times slower than LP depending on the filter order.
4. There are some WLMS modifications to make it comparable in the speed with LP.
5. The Remez exchange algorithm is the most sophisticated for programming while the WLMS is the simplest one.
6. The Remez exchange algorithm is the best choice for designing high-order linear-phase SAW bandpass filters.

Note. The parameter reduction schemes in the frequency or time domain must be applied in LP, WLMS and NLP resulting in the sub-optimum solution.

Part 4: SAW Filter Optimization

SAW Filter Optimization

Basic Assumptions:

1. A SAW filter to be designed consists of two linear phase SAW transducers.
2. Frequency response $F_1(\omega)$ of one of the input SAW transducer is supposed to be given a priori while the frequency response of the output SAW transducer $F_2(\omega)$ is optimized providing a Chebyshev (minimax) approximation of the desired magnitude shape function $D(\omega)$.
3. There are no constraints imposed on the magnitude shape function $D(\omega)$. It may be symmetric, non-symmetric, multi-passband, etc.

Note: A particular case where a SAW filter consists of two apodized SAW transducers giving an optimum overall response (factorizational synthesis) is not treated here.

Multiplicative Chebyshev Approximation Problem

For SAW filter optimization, a weighted error function $E(\omega)$ can be written in the form

$$E(\omega) = W(\omega)\left[D(\omega) - F(\omega)\right] \tag{4.1}$$

$$F(\omega) = \xi(\omega) F_1(\omega) F_2^*(\omega) \tag{4.2}$$

$W(\omega) > 0$	weighting function (Chebyshev)
$F(\omega)$	SAW filter response
$\xi(\omega)$	skewing (shape distorting) factor
$F_{1,2}(\omega)$	array functions of the input and output SAW transducers

The skewing factor $\xi(\omega)$ accounts for SAW transducer element factor, frequency response of a multistrip coupler, etc.

1. A feature of the approximation problem (4.1-4.2) is the multiplicative approximating function $F(\omega)$ (4.2), with the function $F_1(\omega)$ sign-alternating, in general case.
2. The McClellan's computer program [2] implementing the Remez exchange algorithm cannot be directly applied, except for the particular case $\xi(\omega)F_1(\omega)=1$.

Optimum Solution

Auxiliary problem with the modified error function

$$\widetilde{E}(\omega) = sign\{F_1(\omega)\}\, E(\omega) = \widetilde{W}(\omega)\left[\widetilde{D}(\omega) - F_2(\omega)\right], \quad \omega \in \widetilde{\Omega} \qquad (4.3)$$

$$\widetilde{W}(\omega) = \left|\xi(\omega) F_1(\omega)\right| W(\omega), \qquad \widetilde{D}(\omega) = \frac{D(\omega)}{\left|\xi(\omega) F_1(\omega)\right|} \qquad (4.4)$$

The auxiliary approximation problem (4.3-4.4) can be solved on the subset $\Omega'=\{\omega \in [0, \omega_\pi], \omega \neq \omega_i\}$ by any linear Chebyshev approximation method (REA, LP, WLMS), with the initial functions $W(\omega)$ and $D(\omega)$ transformed to the auxiliary functions according to Eq. (4.4).

1. The optimum Chebyshev approximation of the multiplicative problem has a unique optimum solution which does not depend on the optimization technique.

2. The multiplicative approximating function $F(\omega)$ results in additional interesting features of the optimum solution following from the Chebyshev alternation theorem.

Optimum Solution Features

Error function $\widetilde{E}(\omega)$ - n_2+1 equiripple extremuma over Ω (alternation theorem).

Overall function $F(\omega)=\xi(\omega)F_1(\omega)F_2(\omega)$ - $n=n_1+n_2-1>n_2$ extremuma (polynomial product).

This allows for some smaller extremuma to appear in addition to the alternation ones.

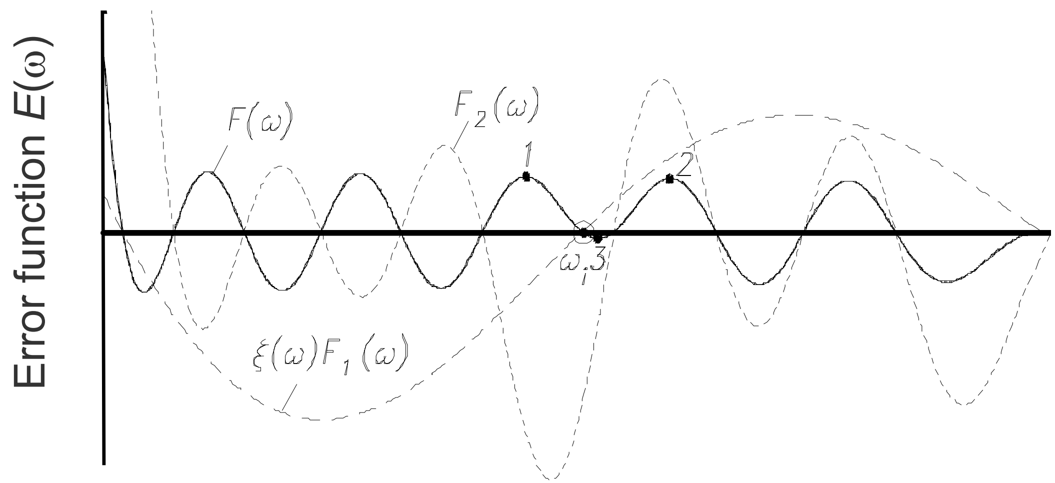


Fig. 4.1. Optimum error function $E(\omega)$ and the arrangement of zeros and extremuma of the functions $\xi(\omega)F_1(\omega)$ and $F_2(\omega)$

Optimum Solution Features (Cont'd)

1. The difference between the maximum extremuma number $n > n_2$ and the alternation extremuma number n_2 forces the extra smaller extremuma to appear between two neighbour equiripple ones .

2. It is a stopband allocation of zeros and extremuma of the functions $F_1(\omega)$ and $F_2(\omega)$ that ensures a solution optimality, with extremuma located in the neighborhood of zeros and vice versa.

Example 1 (REA): SAW Filter Optimization

SAW Filter Specifications

Fractional bandwidth at -3 dB		25 %
Shape factor	-3/-40 dB	1.25
Stopband attenuation		-60 dB
Passband ripple (peak-to-peak)		0.1 dB

Note. The overall skewing function $\xi(\omega)=\omega\zeta^2(\omega)$ comprising the skewing frequency factor ω and the gap-weighted element factor $\zeta(\omega)$ for the metallization ratio of 0.5 was compensated for within a flat filter pass band.

Example 1 (REA): Optimization Parameters

Problem: Multiplicative approximation (4.1), (4.2)

Solution: Variables substitution (4.3), 4.4)

Method: Remez exchange algorithm **(**McClellan's program)

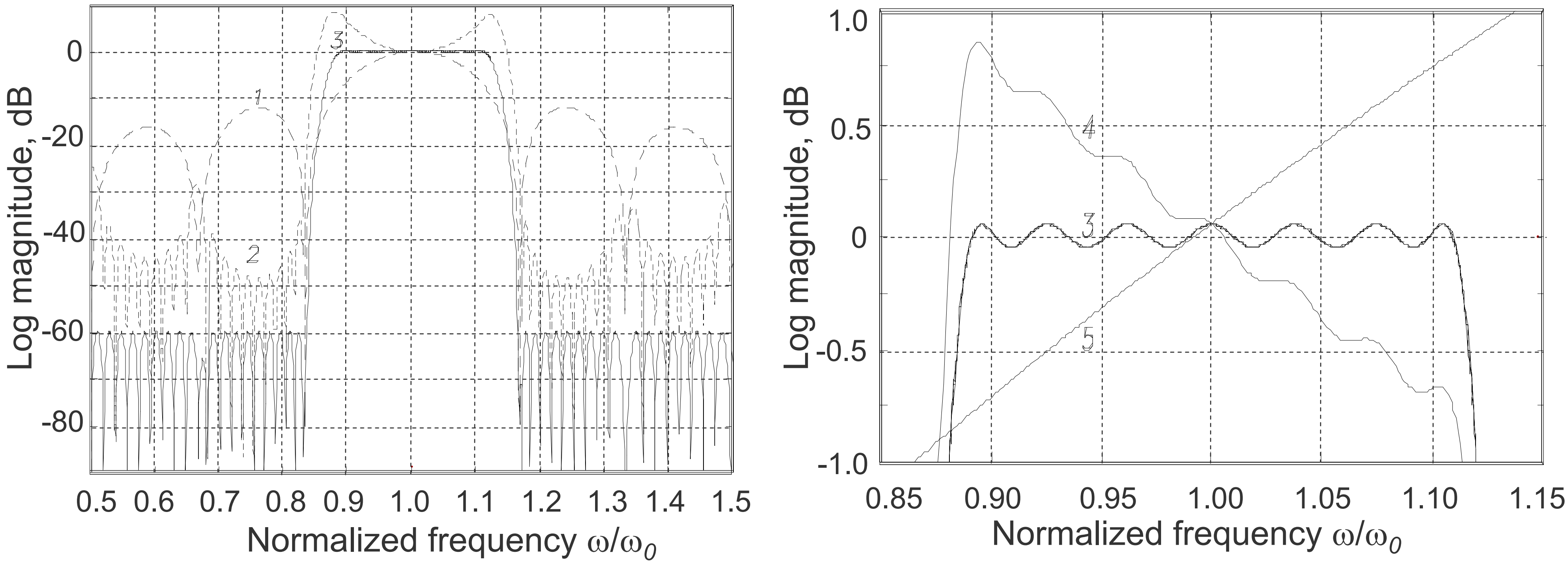
Filter Order: N_1=24 and N_2=200

Synchronism: $\omega_\pi=2\omega_0$ (split finger SAW transducers), ω_0 – central frequency

Discretization: discrete frequency grid *N*g=943 points,
discretization step $\delta\omega = 0.1\Delta\omega$
$\Delta\omega = 2\omega_\pi / N_2$ - frequency sampling interval

Convergence: The number of iterations to attain an optimal solution 110.

Example 1 (REA): Filter Frequency Response



a) Magnitude response

b) Passband response

Fig. 4.2. Optimum SAW filter design:

1 - unapodized IDT response $\sqrt{\xi(\omega)}F_1(\omega)$

2 - apodized IDT response $\sqrt{\xi(\omega)}F_2(\omega)$

3 - overall response $\xi(\omega)F_1(\omega)F_2(\omega)$

4 - polynomial product $F_1(\omega)F_2(\omega)$

5 - skewing factor $\xi(\omega) = \omega\xi^2(\omega)$

Example 2 (WLMS): TV SAW Filter

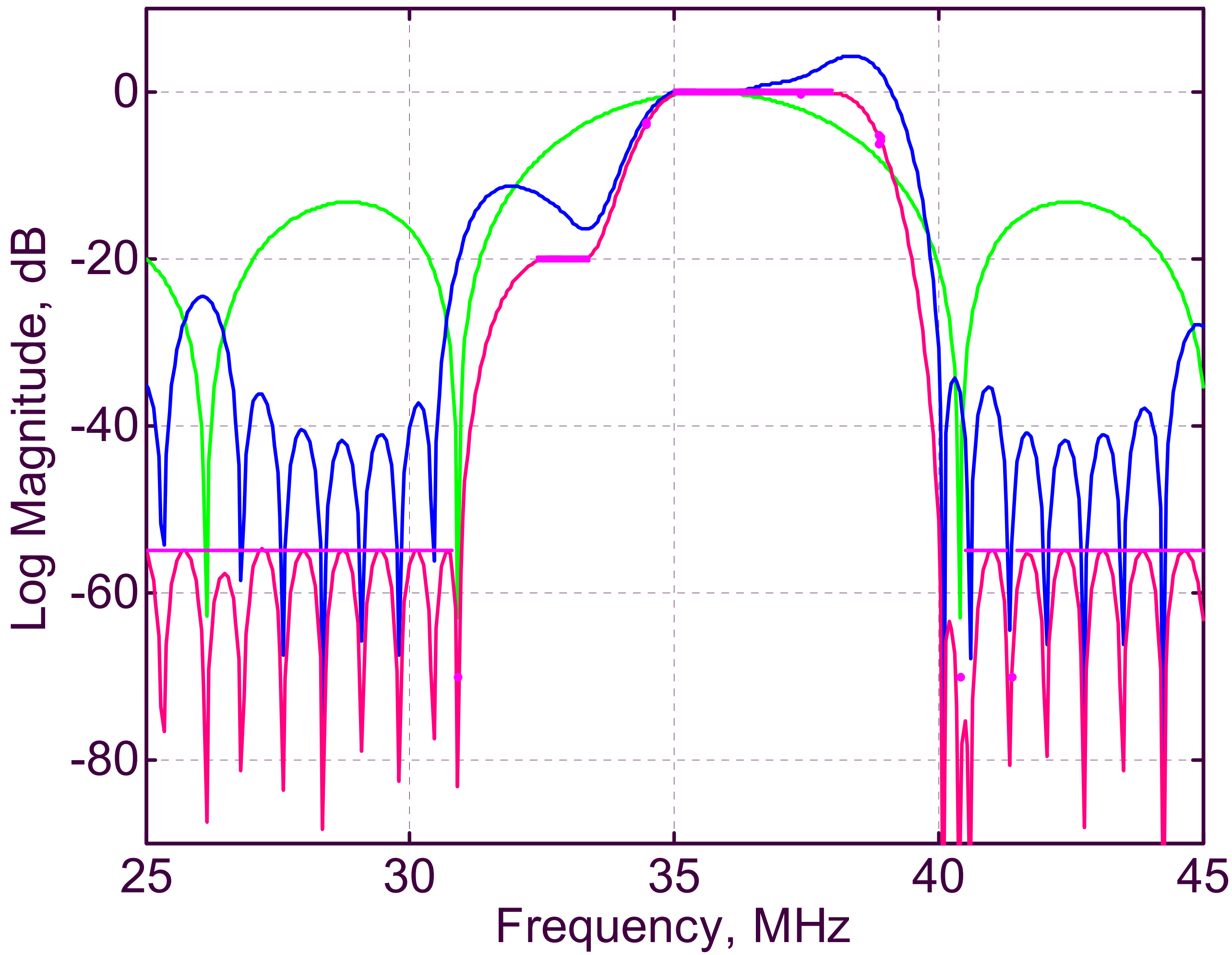


Fig. 4.3. WLMS design, TV SAW Filter with prescribed magnitude response, N_1=32, N_2=170, - - - input, —•— output, —— overall response

Example 2 (WLMS): SAW Filter (Passband)

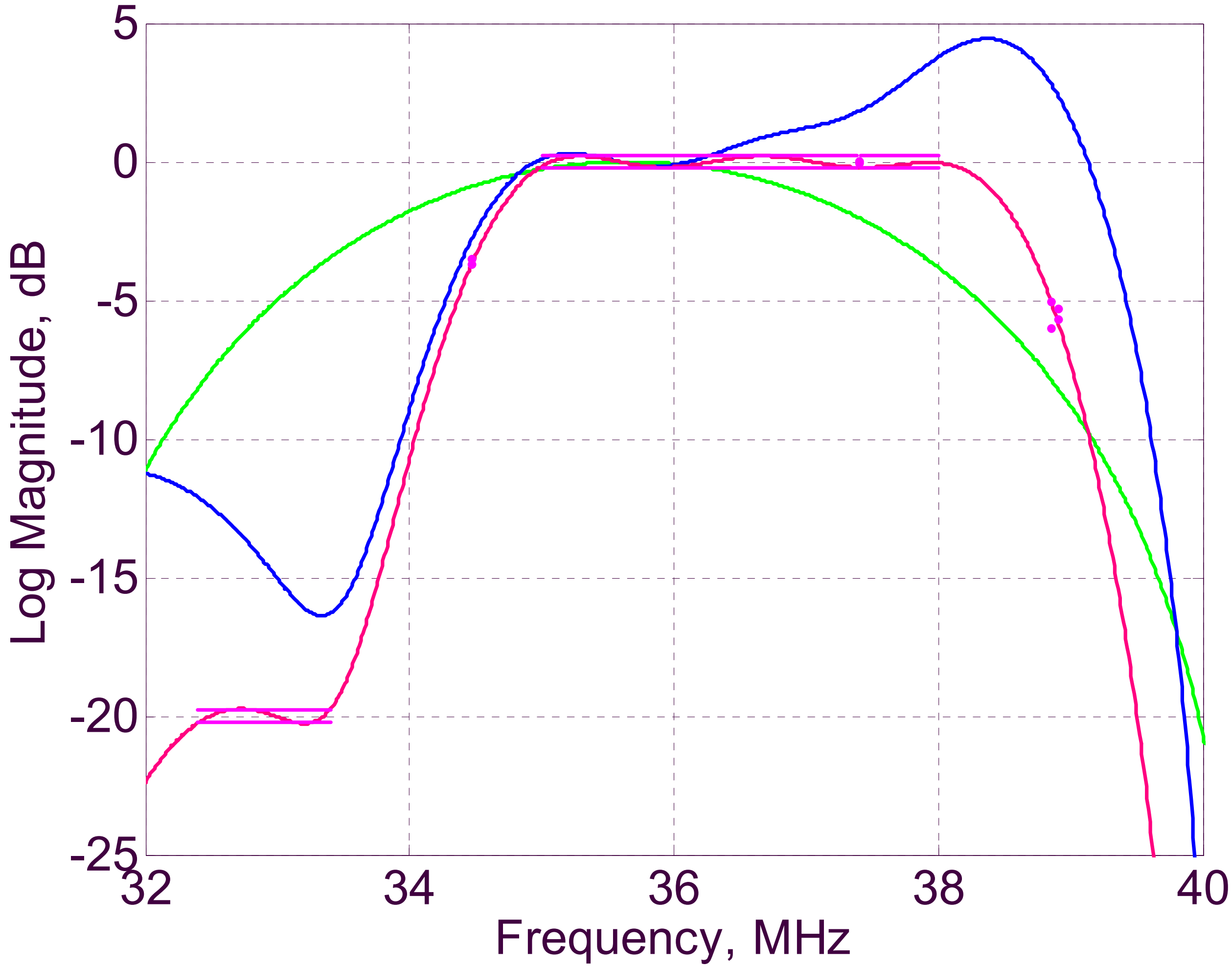


Fig. 4.4. TV SAW Filter with prescribed magnitude response (pass band),

N_1=32, N_2=170, - - - input, — • — output, —— overall response

Example 2 (WLMS): SAW Filter (Tap Weights)

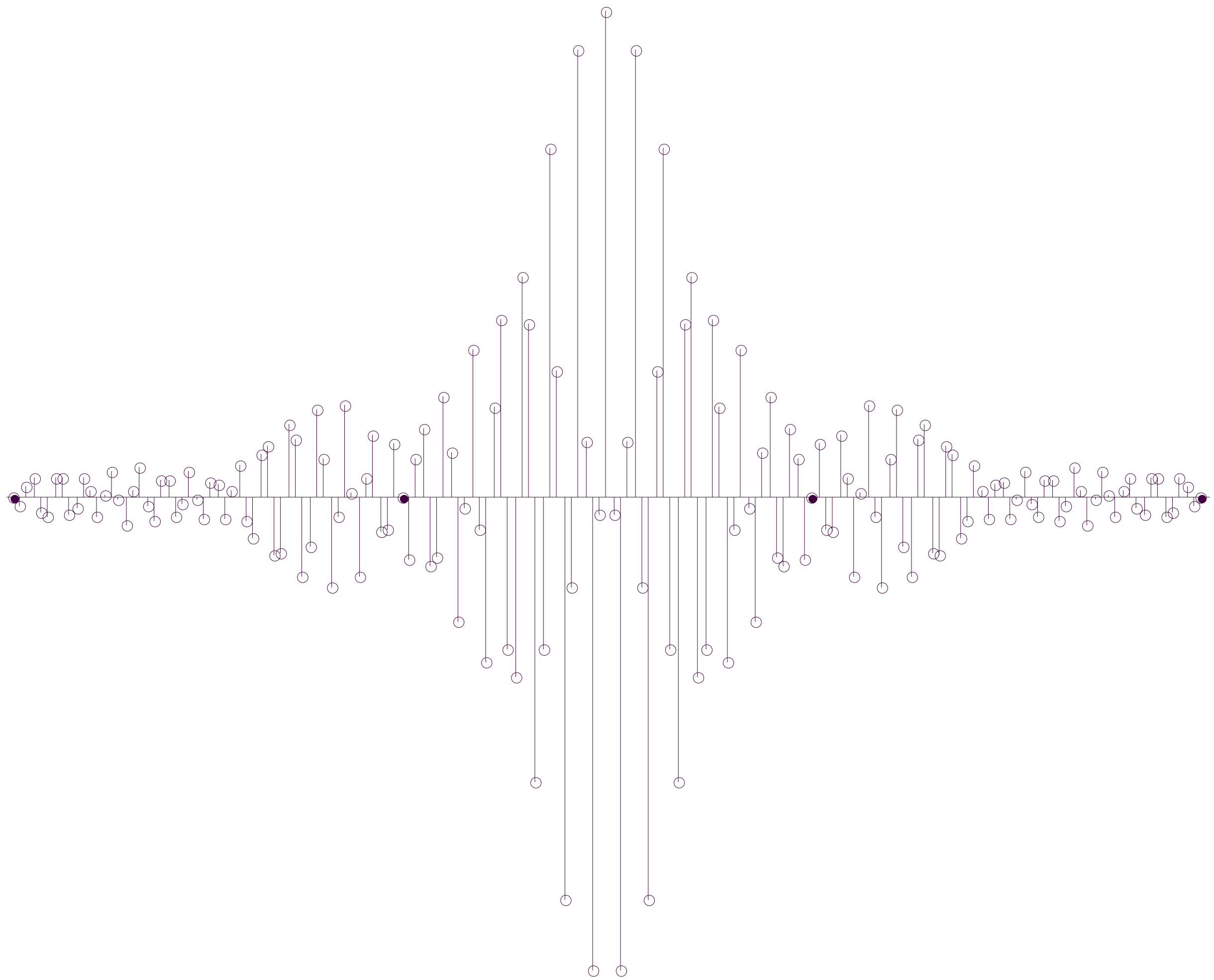


Fig. 4.5. TV SAW Filter tap weights (apodized SAW transducer), N=170

Frequency Response Properties

Trigonometric Representation

$$F(\varphi) = C(\varphi) + jS(\varphi) = \sum_{k=0}^{N-1} A_k e^{j(k-\frac{N-1}{2})\varphi}, \quad \varphi = \pi\omega / \omega_\pi \tag{4.5}$$

$$C(\varphi) = \mathrm{Re}\{F(\varphi)\} = \begin{cases} \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos k\varphi, & N = 2n+1 \\ \sum_{k=1}^{n} a_k \cos(k - \frac{1}{2})\varphi, & N = 2n \end{cases} \tag{4.6}$$

$$S(\varphi) = \mathrm{Im}\{F(\varphi)\} = \begin{cases} \sum_{k=1}^{n} b_k \sin k\varphi, & N = 2n+1 \\ \sum_{k=1}^{n} b_k \sin(k - \frac{1}{2})\varphi, & N = 2n \end{cases} \tag{4.7}$$

Linear-Phase Property

The functions $C(\varphi)$ and $S(\varphi)$ are the trigonometric polynomials with *cosine* and *sine* basis functions and the arguments which depends on whether the tap number is even or odd.

Relationship between tap weights A_k and trigonometric polynomial coefficients a_k and b_k

$$A_{n\pm k} = \frac{a_k \pm b_k}{2}, \quad k = (0), 1, \ldots, n \qquad (4.8)$$

1. In general case, frequency response is a complex-valued Hermitian function

$$F(\varphi)=F^*(-\varphi). \qquad (4.9)$$

2. SAW transducer (or FIR digital filter) has an ideal linear phase at all frequencies in two particular cases:
   a) symmetric tap weight distribution $A_{n-k}= A_{n+k}$, $F(\varphi)=C(\varphi)$
   b) antisymmetric tap weight distribution $A_{n-k}=-A_{n+k}$, $F(\varphi)=jS(\varphi)$

Symmetry and Periodicity

N	$F(\omega$	$f_k(\omega)$	Period	$F(\omega_i)=0$	Pass Band	ω_0	Taps
$2n+1$	C_{2n+1}	$\cos k\varphi$	$2\omega_\pi$	-	1) Low pass 2) Band pass 3) High pass	$\omega_0=0$ $0<\omega_0<\omega_\pi$ $\omega_0\leq\omega_\pi$	- ΔV_k ΔV_k, V_k
	S_{2n+1}	$\sin k\varphi$	$2\omega_\pi$	$n\omega_\pi$	Band pass	$0<\omega_0<\omega_\pi$	ΔV_k
$2n$	C_{2n}	$\cos (k-½)\varphi$	$4\omega_\pi$	$(2n+1)\omega_\pi$	1) Low pass 2) Band pass	$\omega_0=0$ $0<\omega_0<\omega_\pi$	- V_k
	S_{2n}	$\sin (k-½)\varphi$	$4\omega_\pi$	$2n\omega_\pi$	1) Band pass 2) High pass	$0<\omega_0<\omega_\pi$ $\omega_0\leq\omega_\pi$	V_k ΔV_k, V_k

LP - low pass, BP - band pass, HP - high pass

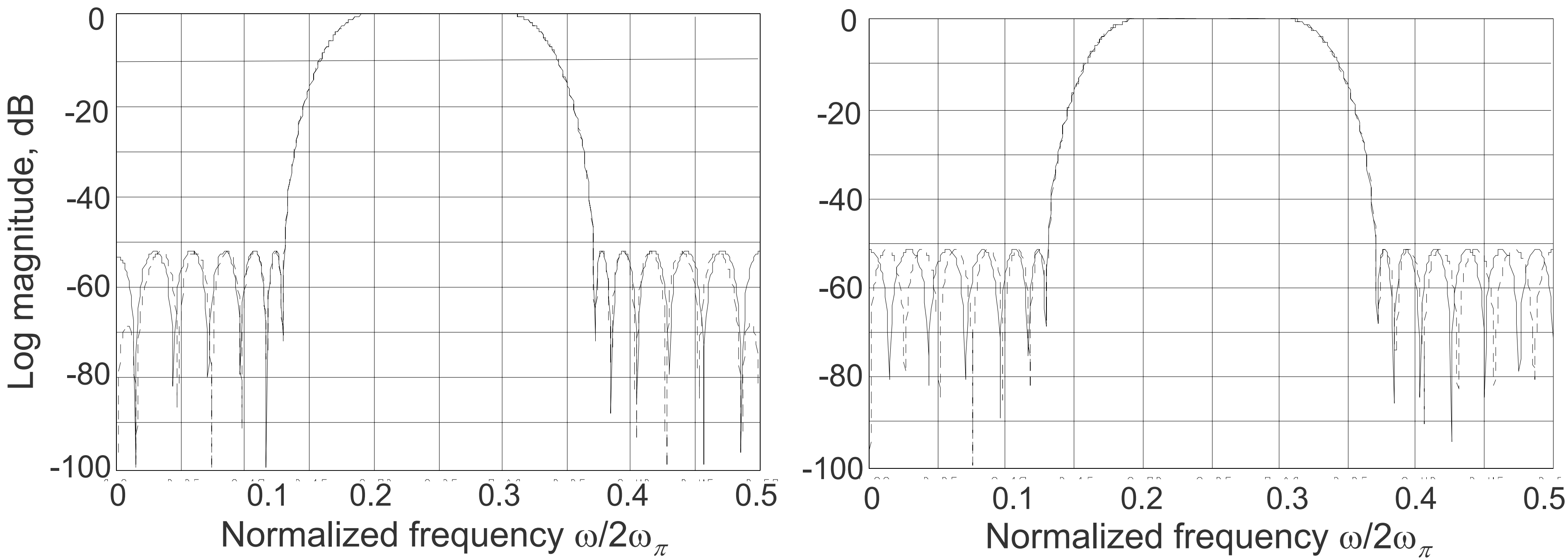
$\omega_\pi=\pi v/p$ - synchronous frequency (v – SAW velocity, p – period (pitch)

$\varphi=\pi\omega/\omega_\pi$ - phase lag per period

Basic Properties

1. Symmetry and periodicity properties of the function $F(\omega)=C(\omega)+S(\omega)$ depend on the number of taps $N=2n+1$ (odd) or $N=2n$ (even) that uniquely defines the trigonometric basis.
2. There exist restrictions on SAW transducer design due to the singularity points at the frequencies $m\omega_\pi$ where the frequency response may be identically equal to zero for particular frequency types.
3. The characteristics $S_{2n+1}(\omega)$ and $S_{2n}(\omega)$ vanish at frequencies $m\omega_\pi$. They cannot be used for LP designs.
4. The functions $S_{2n+1}(\omega)$ and $C_{2n}(\omega)$ vanish at frequencies $(2m+1)\omega_\pi$. They cannot be used for HP filters (solid (unsplit) SAW transducers).
5. Any type of the frequency response is appropriate for designing asynchronous FIR filters or SAW transducers with $\omega_0 \neq \omega_\pi$ having more than two taps per period in general case.
6. The functions $C_{2n+1}(\omega)$ and $S_{2n+1}(\omega)$ are appropriate for overlap- weighting and $C_{2n}(\omega)$ and $S_{2n}(\omega)$ for finger-weighting.

Linear-Phase Frequency Response



a) odd number of taps N=37

b) even number of taps N=36

Fig. 4.6. FIR filter (SAW transducer) linear phase response: —— $C(\omega)$, - - - $S(\omega)$

Tap Weights

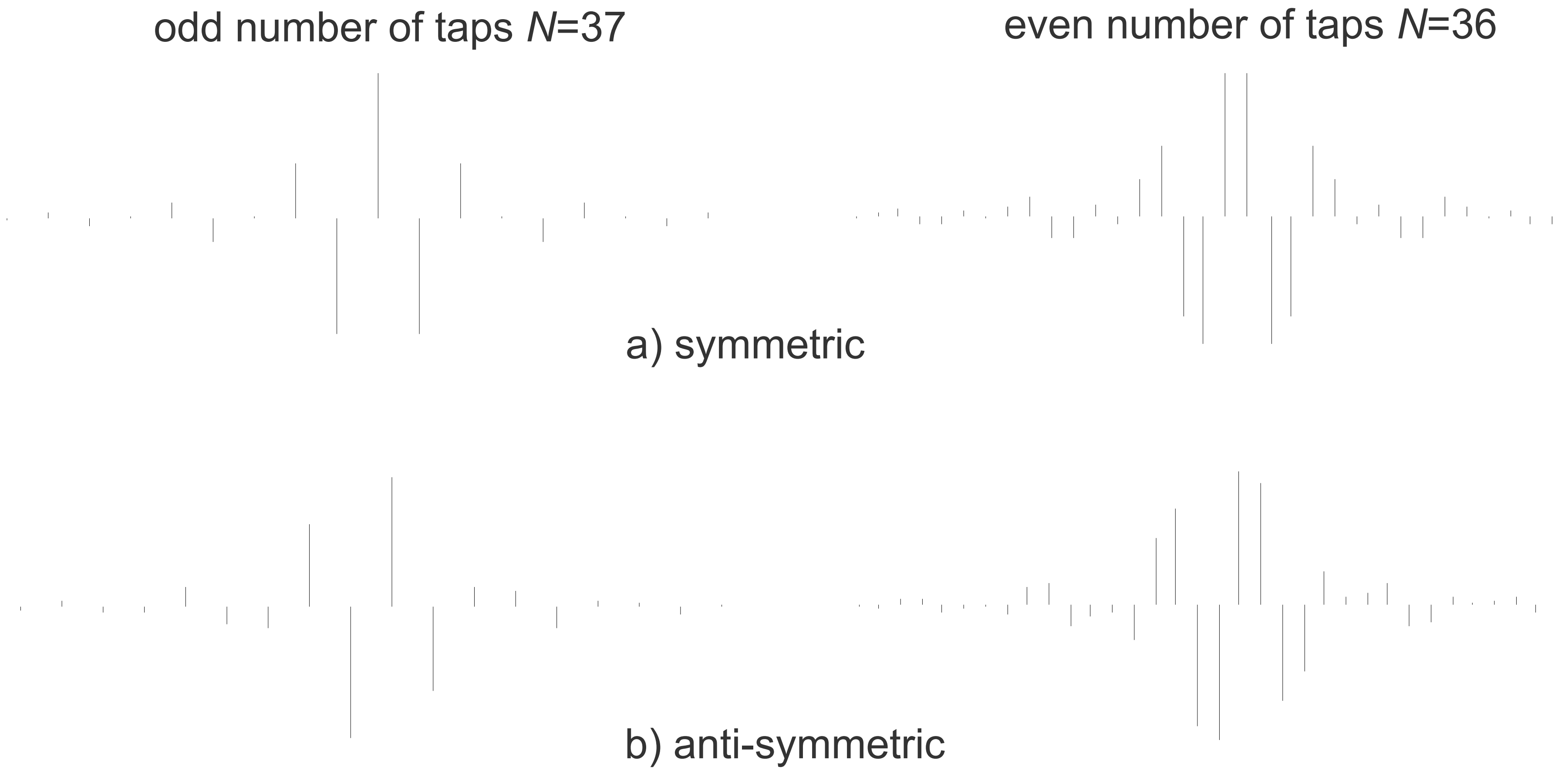


Fig. 4.7. Discrete time response (tap weights) of the linear phase SAW transducer

Part 5: Suboptimum Design: Parameter Reduction

Frequency Sampling

Frequency response (referenced to transducer center)

$$F(\varphi) = C(\varphi) + jS(\varphi) = \sum_{k=0}^{N-1} A_k e^{jk\varphi} \quad (5.1)$$

Discrete version of Eq. (5.1)

$$F(\varphi_i) = C(\varphi_i) + jS(\varphi_i) = \sum_{k=0}^{N-1} A_k e^{jk\varphi_i} \quad (5.2)$$

$$\varphi_i = i\Delta\varphi, \quad \Delta\varphi = \frac{2\pi}{N}, \quad i = \overline{0, N-1}$$

Coefficients (tap weights) A_k

$$A_k = \frac{1}{N}\sum_{i=0}^{N-1} F(\varphi_i) e^{-j(k-\frac{N-1}{2})\varphi_i}, \quad k = \overline{0, N-1} \quad (5.3)$$

Frequency Sampling Interpolation

Interpolation:

$$F(\varphi) = \sum_{k=0}^{N-1} F(\varphi_i)\,\mathrm{sinc}(\varphi - \varphi_i) \tag{5.4}$$

Basis Functions:

$$\mathrm{sinc}(\varphi - \varphi_i) = \frac{1}{N} \frac{\sin \frac{N}{2}\varphi}{\sin \frac{1}{2}\varphi} \tag{5.5}$$

Symmetry Relations:

$$F(2\pi - \varphi_i) = \begin{cases} F^*(\varphi_i), & N = 2n+1 \\ -F^*(\varphi_i), & N = 2n \end{cases} \tag{5.6}$$

$$\mathrm{sinc}(\varphi - \varphi_i + 2\pi) = \begin{cases} \mathrm{sinc}(\varphi + \varphi_i), & N = 2n+1 \\ -\mathrm{sinc}(\varphi + \varphi_i), & N = 2n \end{cases} \tag{5.7}$$

Simplifications

We can rewrite Eq. (5.4) in the following equivalent form

$$F(\varphi) = \sum_{k=0}^{n} F(\varphi_i)\,\mathrm{sinc}(\varphi - \varphi_i) + \sum_{k=0}^{n} F^*(\varphi_i)\,\mathrm{sinc}(\varphi + \varphi_i) \qquad (5.8)$$

or by separating the real and imaginary parts

$$C(\varphi) = \sum_{k=0}^{n} C(\varphi_i)\left\{\mathrm{sinc}(\varphi - \varphi_i) + \mathrm{sinc}(\varphi + \varphi_i)\right\} \qquad (5.9)$$

$$S(\varphi) = \sum_{k=0}^{n} S(\varphi_i)\left\{\mathrm{sinc}(\varphi - \varphi_i) - \mathrm{sinc}(\varphi + \varphi_i)\right\} \qquad (5.10)$$

Basis functions in (56-58) are periodic versions of sin*X*/*X* functions

$$\mathrm{sinc}(\varphi) = \frac{1}{N}\frac{\sin\frac{N}{2}\varphi}{\sin\frac{1}{2}\varphi} \approx \frac{1}{N}\frac{\sin\frac{N}{2}\varphi}{\frac{1}{2}\varphi} = \frac{\sin X}{X}, \quad X = \frac{N}{2}\varphi \qquad (5.11)$$

Functions sinc(φ-φ_i) account for a contribution to the frequency response $F(\varphi)$ of the main pass band samples $F(\varphi$-$\varphi_i)$ while functions sinc(φ+φ_i) account for a contribution of the "mirror" pass band that can be neglected in many cases.

Basis Functions: Odd N

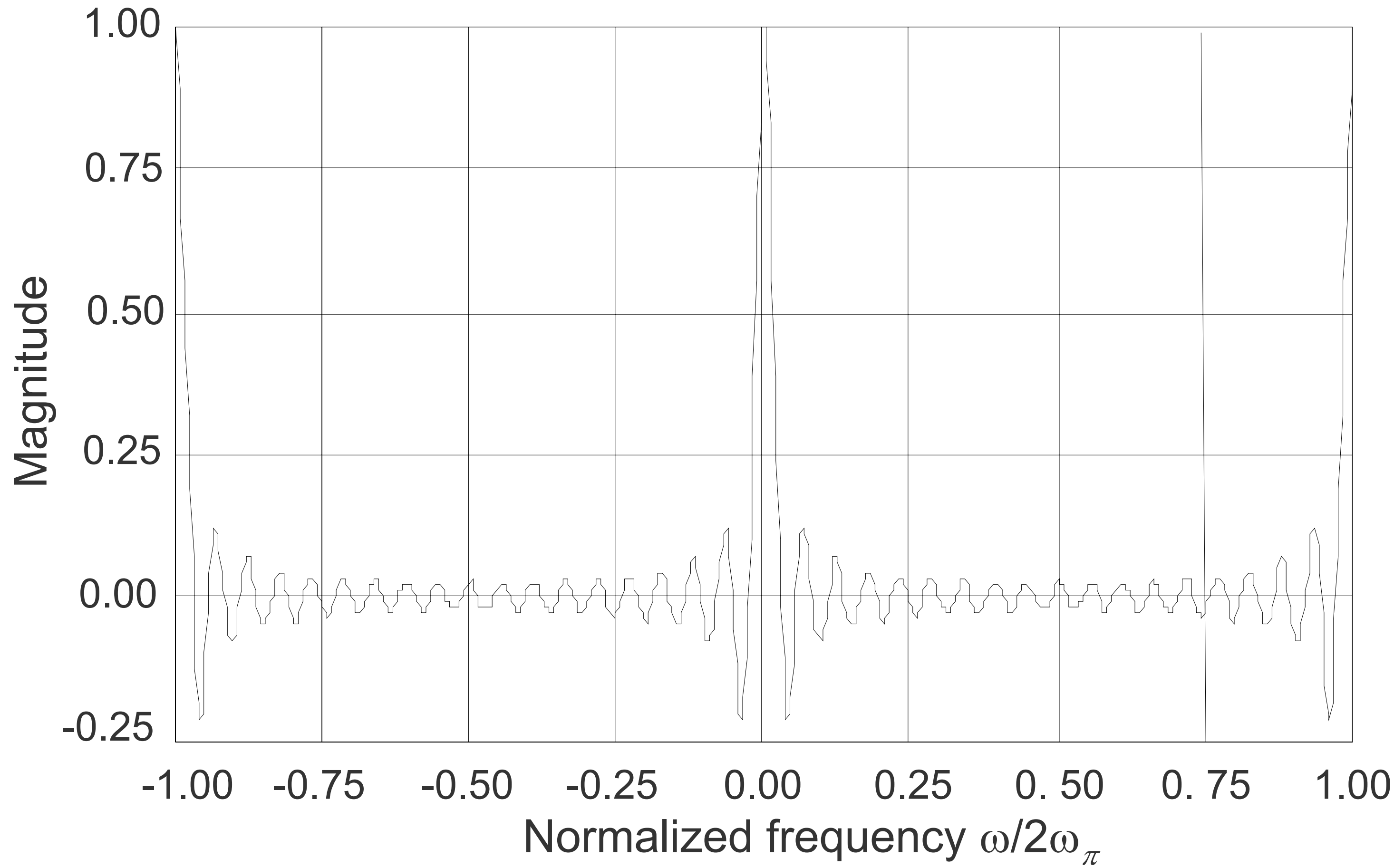


Fig. 5.1. Basis functions *sinc*(φ) for odd number of taps N=37

Basis Functions: Even *N*

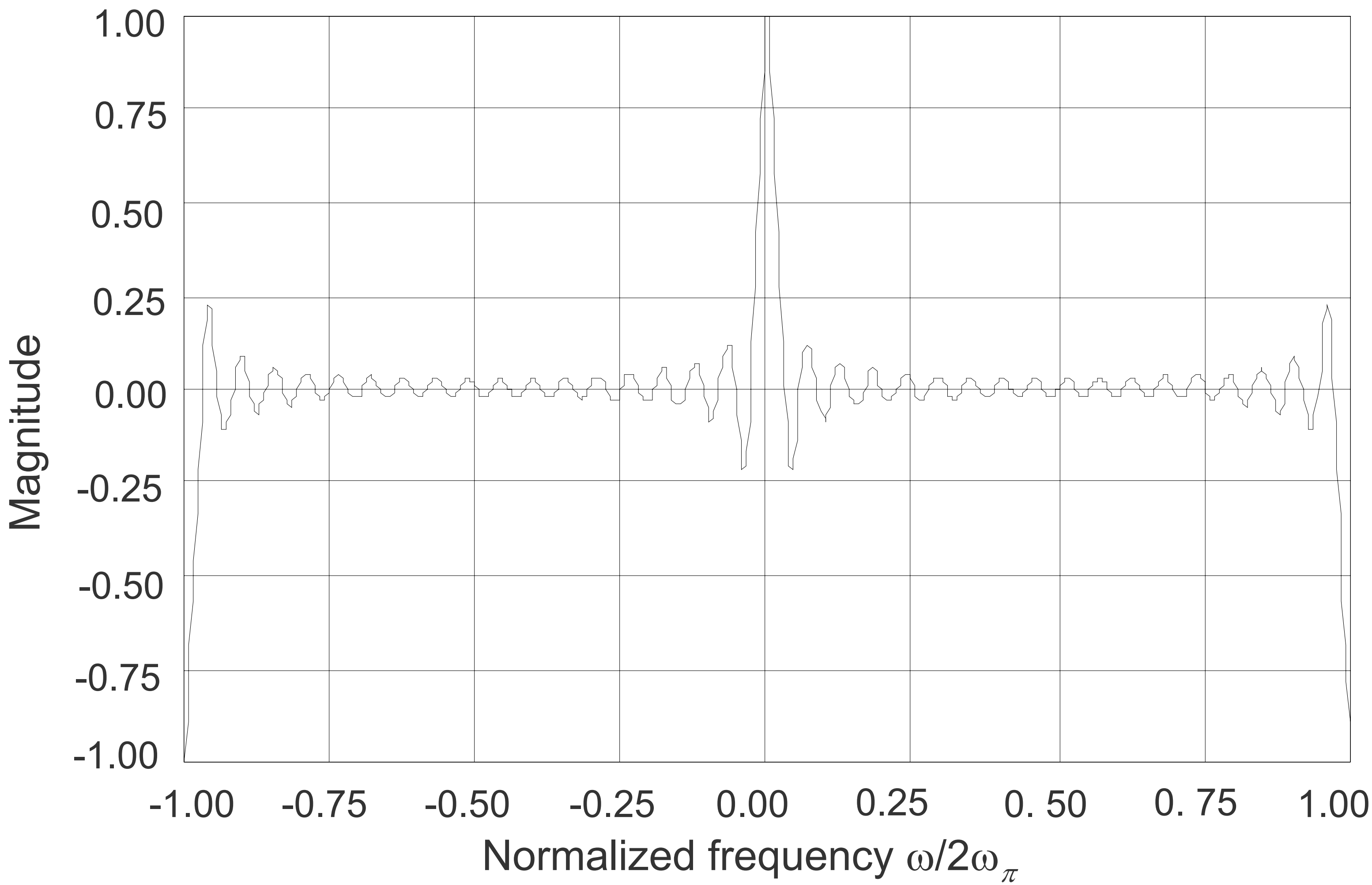


Fig. 5.2. Basis functions *sinc*(φ) for even number of taps *N*=37

Frequency Sampling: Hamming Window Function

$$F_0 = 1.08,\ F_{\pm 1} = 0.46 \Rightarrow a_k = 0.54 + 0.46\cos\frac{2\pi k}{N}$$

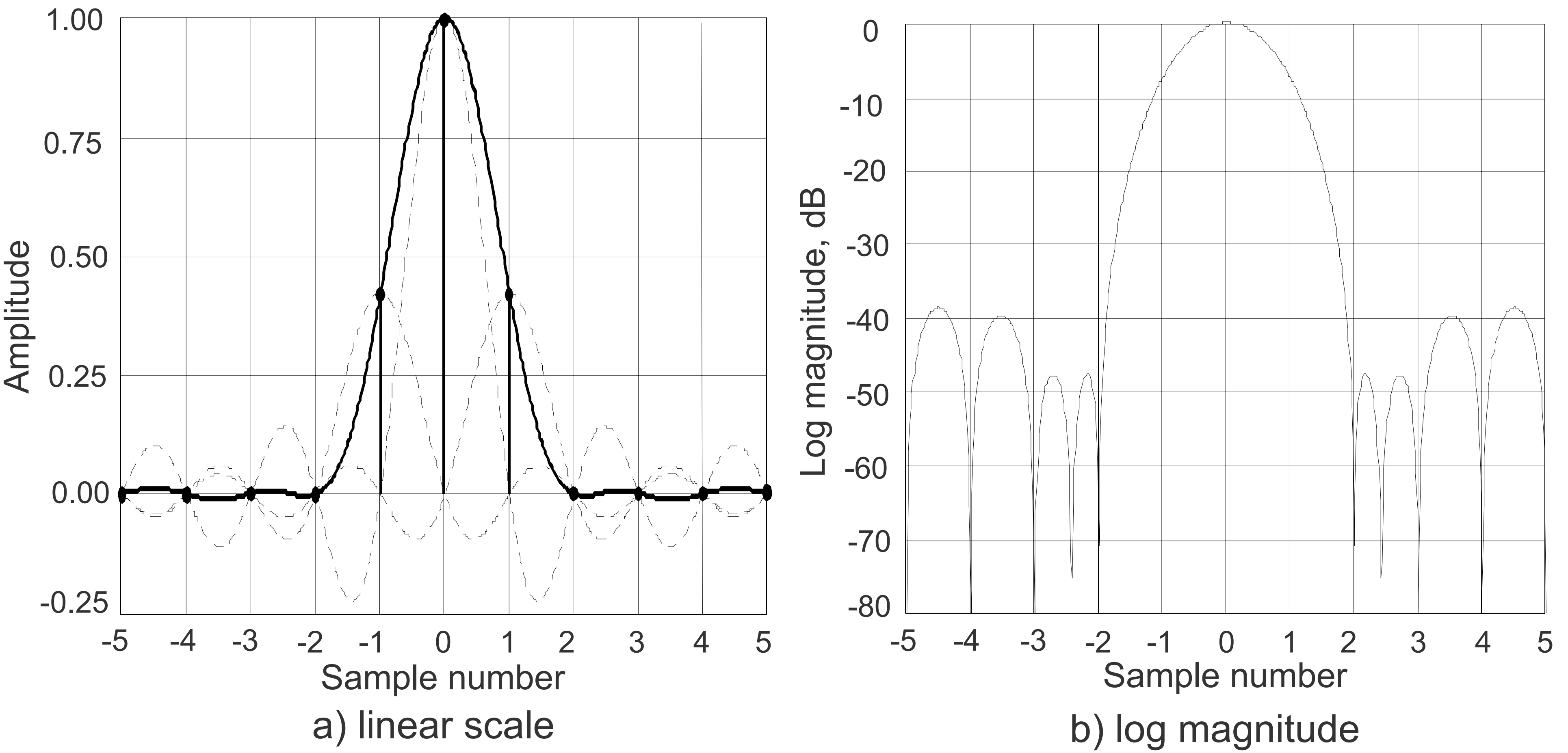


Fig. 5.3. Frequency sampling interpolation $F(\varphi)$: Hamming window function, N=20

Reduction of Frequency Samples Number

Observation: Major contribution to the frequency response is given by the relatively small fraction of the frequency samples located in the pass band.

Corollary: Most of the far-field samples may be set to zero values without significant sacrificing of the approximation accuracy.

Effect: Slight deterioration of the out-of-band rejection while the magnitude shape and pass band ripple are unaffected, provided for the high out-of-band rejection.

1. The frequency sampling technique excludes the redundancy in the number of the variables inherent to optimum design.
2. The frequency sampling allows to reduce considerably the number of the optimized variables without sacrificing the approximation accuracy.
3. The minimum number of frequency samples does not depend on the filter central frequency ω_0.
3. The optimization with reduced set of samples (optimized variables **)** gives a suboptimum solution, with the computational time considerably reduced.
4. The frequency sampling method perfectly suits to LP and WLMS optimization.

Frequency Samples Contribution (Stop Band)

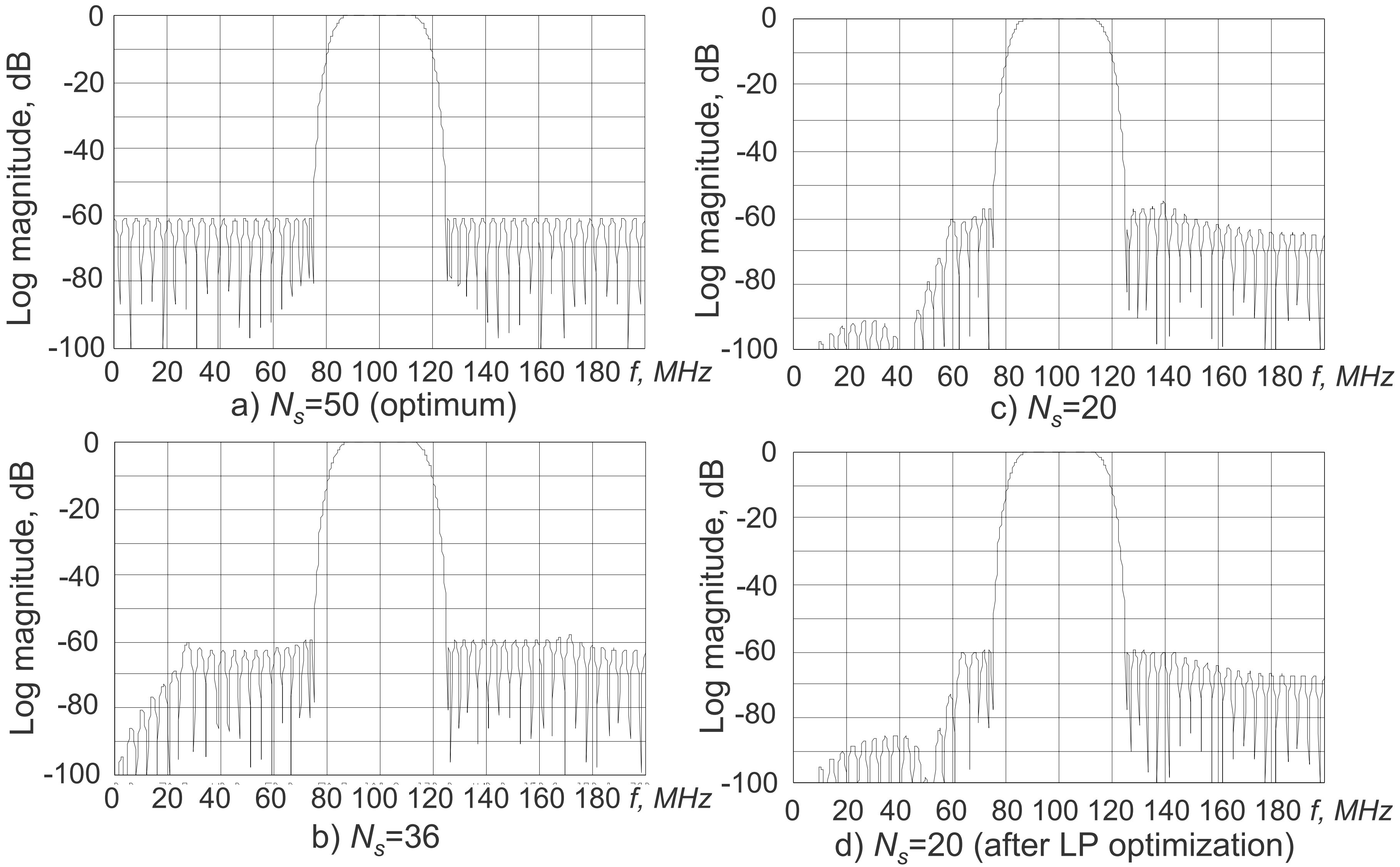


Fig. 5.5. Optimum and suboptimum frequency response (N=101)

Frequency Samples Contribution (Pass Band)

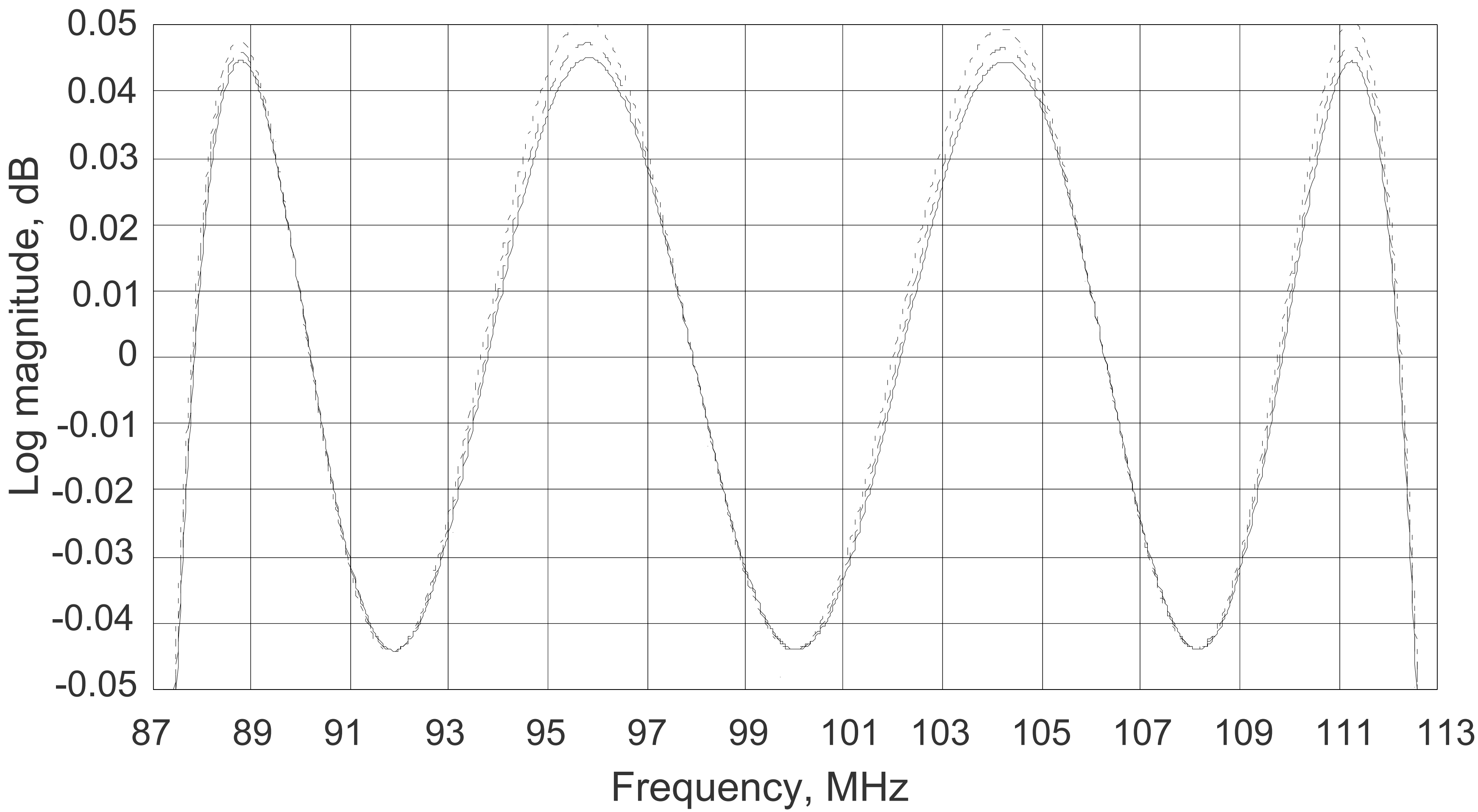


Fig. 5.6. Passband frequency response for different number of the non-zero frequency samples: —— N_s=50 (optimum), — — N_s=36, - - - N_s=20, · · · N_s=20 (LP optimized)

Time Domain Downsampling and Interpolation

Base Band ⟷ Band Pass

Time

t

For the narrowband band limited signal, the bandpass time (impulse) response $h(t)$ can be represented in the form

$$h(t) = \hat{h}_c(t)\cos\omega_0 t - j\hat{h}_s(t)\sin\omega_0 t \qquad (5.12)$$

where ω_0 is the central frequency and the function

$$\hat{h}(t) = \hat{h}_c(t) + j\hat{h}_s(t) \qquad (5.13)$$

is the base band time response, with the real functions $\hat{h}_c(t)$ and $\hat{h}_s(t)$ being the in-phase and quadrature signal components, respectively

Time Domain Downsampling and Interpolation (Cont'd)

Frequency

Base Band $\Longleftrightarrow$ **Band Pass**

ω

Fourier transform of Eq. (53)

$$F(\omega) = \frac{1}{2}\widehat{F}(\omega - \omega_0) + \frac{1}{2}\widehat{F}(\omega + \omega_0) \approx \frac{1}{2}\widehat{F}(\omega - \omega_0) \qquad (5.14)$$

$\widehat{F}(\omega) = \mathcal{F}\left\{\hat{h}(t)\right\}$ base band (zero frequency) frequency response

$F(\omega) = \mathcal{F}\left\{h(t)\right\}$ band pass frequency response

Shannon's Sampling Theorem

Supposed for the base band frequency response $\hat{F}(\omega)$ to be band limited of the width Ω, the base band time response can be interpolated as follows

$$\hat{h}(t) = \sum_{k=-N_s/2}^{N_s/2} \hat{h}(t_k) \frac{\sin \frac{\Omega}{2}(t - t_k)}{\frac{1}{2}(t - t_k)}, \quad t_k = k\Delta T, \quad \Delta T = \frac{2\pi}{\Omega} \qquad (5.15)$$

Frequency and Time Domain Duality

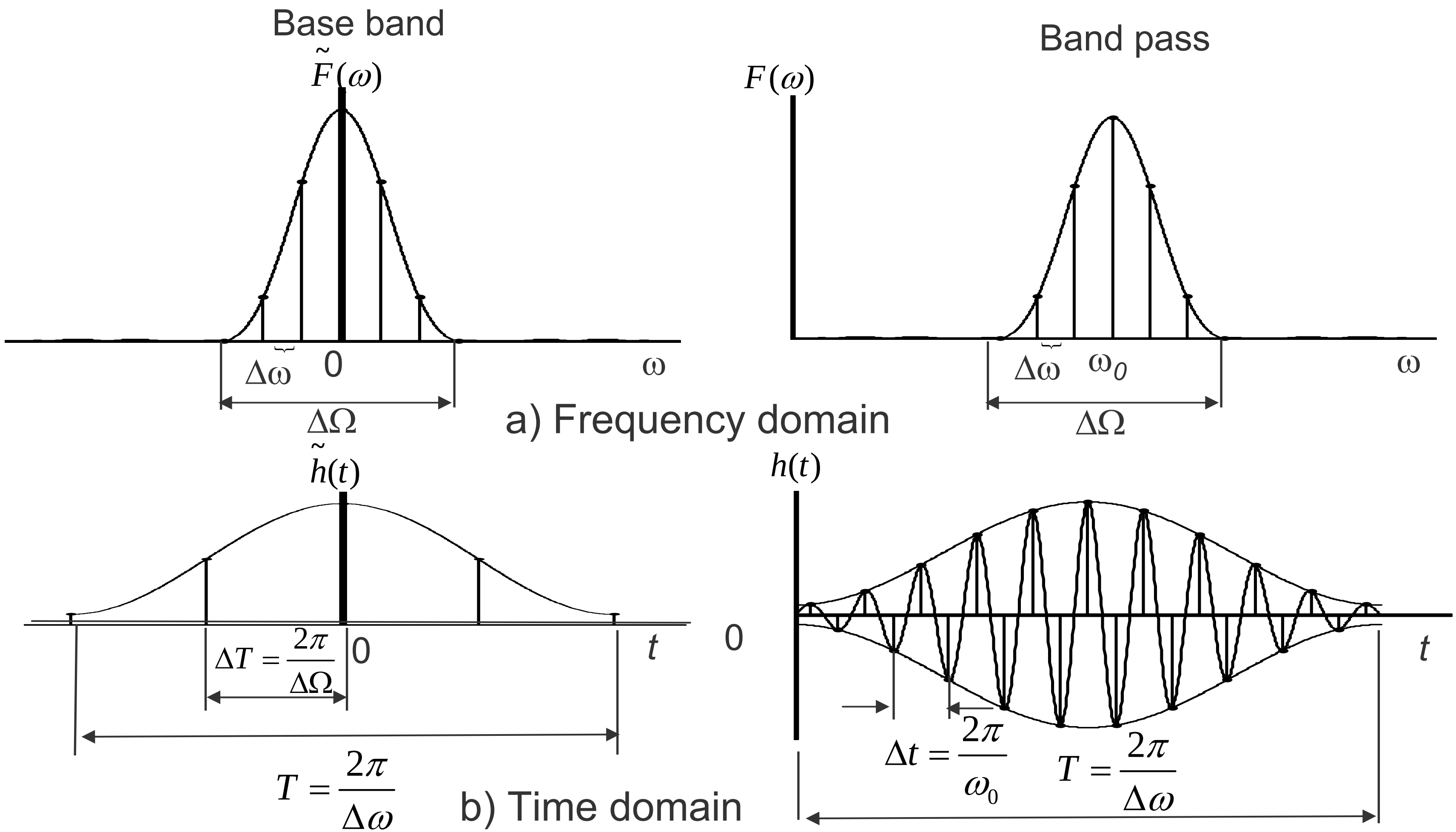


Fig. 5.7. Frequency and time domain interpolation of the base band and band pass response

Number of Frequency and Time Domain Samples

The length T of the time response is defined by the frequency sampling interval $\Delta\omega$

$$T = \frac{2\pi}{\Delta\omega} \tag{5.16}$$

Number of the base band time response samples $\hat{h}(t_k)$

$$N_s = \frac{T}{\Delta T} = \frac{2\pi / \Delta\omega}{2\pi / \Omega} = \frac{\Omega}{\Delta\omega} \tag{5.17}$$

Minimum number of the frequency or time samples is the same, for the band limited frequency response.

Narrowband condition: $\Omega << \omega_0 \Longrightarrow N_s << N$

where $N=2\omega_0/\Delta\omega$ is the number of the DFT coefficients A_k, $k=0, N-1$.

1. In time domain, the pass band to base band cobersion allows to considerably reduce number of the independent variables.
2. The gain in the number of the optimized variables (time or frequency samples) is the same both in time and frequency domain.
3. A band pass time response can be restored by applying the Shannon discretization theorem Eq. (5.15) and the base band to band pass conversion Eq. (5.12).

Conclusions: General Properties

- The Chebyshev approximation problem has a unique optimum solution satisfying the alternation theorem.
- The feature of the optimum solution is the equiripple behavior of the error function both in the stopband and passband.
- Three basic techniques for the optimum Chebyshev approximation: Remez exchange algorithm (REA), linear programming (LP), and weighted least-mean squares (WLMS).
- The REA and WLMS are iterational procedures to exploit equiripple behavior.
- LP is the most general and powerful linear optimization technique.
- LP does not use the alternation/equiripple property of the error function and orthogonality of the basis functions.

Conclusions: Computational Speed

- For the same number of the variables, the REA is much superiour over the LP and WLMS.
- LP and WLMS have the same order of the computational speed.
- The principal WLMS advantage is an easy programming if compared to other optimization algorithms.

Conclusions: Parameter Number Reduction

- In practice, parameter reduction schemes in the frequency or time domain should be applied in the LP and WLMS.
- The number of the optimized variables can be reduced both in the frequency and time domains by using the interpolation (decimation) techniques.
- The frequency sampling interpolation allows to reduce the number of the optimized variables (frequency samples) practically with no sacrificing the approximation accuracy.
- All the known linear optimization techniques require adaptation to the SAW filter optimization due to the multiplicative nature of the approximating function.

Conclusions: Suboptimum Design

- While maintaining the optimum synthesis generality and flexibility, the suboptimum synthesis (frequency or time sampling) allows to considerably reduce the number of the optimized variables and hence the storage and the computation time.
- The difference between optimum and suboptimum approximations does not exceed 1-2 dB in the SAW filter stopband and 0.01-0.05 dB in the passband.
- A feature of the frequency sampling interpolation is that the amount of computations is mostly specified by the SAW filter magnitude shape specifications and it does not depend on its central frequency. As a result, a narrowband fast cut-off SAW filters with the large electrode number of several hundred and even thousands can be designed, with the number of the optimized variables drastically reduced.
- The design experience confirms the fast convergence, speed, reliability, and flexibility of the optimum and suboptimum techniques (REA, LP, WLMS).

Conclusions: SAW Filter Design

- The problem is simplified in the assumption that one of the SAW transducers (usually, the unapodized one) is specified a priori, while the other transducer (apodized) is optimized to provide the best fit to the required magnitude response.
- The multiplicative problem of the SAW filter optimization can be converted to the standard form of the linear optimization by modifying the initial weighting and desired (target) functions.
- The more complicated approximating function in SAW filter design results in the particular behavior of the error function, with smaller extremuma appear in the SAW filter stop band.
- It is practical to specify the frequency response of the optimized SAW transducer in terms of the frequency samples to be optimized by LP or WLMS.
- The time sampling technique based on the Shannon theorem for the baseband time response can be applied with the same reduction in the number of the optimized variables as in the frequency sampling technique.

Conclusions : Suboptimum Design

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The End

Thanks for your attention.

Questions?