Tutorial seminar

The magic of cross-correlation in measurements from dc to optics

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The reduction of the instrument background noise is a challenge in numerous domains of experimental science and technology. The correlation method provides a solution in most cases when it is possible to measure the same device under test (DUT) with two separate and independent instruments.

Let us denote with $c(t)$ the physical quantity to be measured, with $a(t)$ and $b(t)$ the background of the two instruments, and with $x(t) = c(t) + a(t)$ and $y(t) = c(t) + b(t)$ the signals available at the instrument outputs. Assuming that the process are stationary and ergodic (the physical experiment is repeatable and reproducible) and that the two instruments are independent, the average correlation of x and y gives the statistical properties of $c(t)$. The single-channel noise is rejected proportionally to the square root of the number m of averages, and ultimately to the square root of the measurement time. The background noise is limited by the thermal inhomogeneity of the system instead of the absolute temperature.

The Wiener-Khinchin theorem guarantees that the average product of the Fourier transform of $x(t)$ and $y(t)$ converges to the power spectrum of $c(t)$. The smoothness of the cross-spectrum tells us whether m is sufficient or not for the cross-spectum to converge to the DUT noise. This property enables to validate the result in the case of AM noise and laser RIN, where we cannot assess the single-channel noise of the instrument without a suitable low-noise reference.

In AM and PM noise measurements, we use correlation with the bridge or the differential method, and the synchronous detection. A background noise of parts in 10^{-21} rad^2/Hz (white) and of 10^{-18} rad^2/Hz (flicker at 1 Hz) has been reported.

Of course the correlation method finds applications in numerous domains. It is the basis of the correlation receiver used in radio-astronomy, with which R. Hanbury-Brown measured the first radio sources in the Cassiopeia and Cygnus constellations. The correlation radiometer followed, opening the way to a future re-definition of the temperature in terms of fundamental constants. Laser beams, chemical batteries and other dc references have been measured with correlation. In semiconductor technology small random signals reveal impurities, defects and energy traps of a dc-biased sample. Another exotic application is the measurement of electromigration in metals at high current density, through the asymmetry between AM and PM $1/f$ noise, which impacts on VLSI technology.

A presentation is available on the author's home page http://rubiola.org [seminar slides].